

# A review on symmetric games: theory, comparison and applications

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**Abstract** Game theory models decision makers' behaviors in strategic situations. Since the structures of games are different, behavior and preferences of the players are different in various types of game. This paper reviews various situations of games. Here, characteristics of some common games are discussed and compared. Specifically, we focus on a group of games called symmetric games including Prisoner's Dilemma, Chicken game, Stag-Hunt and Deadlock. Scenario, outcomes, structure, and equilibrium of these games are explained and compared. Moreover, the applications of these games are discussed to show that they are not just interesting or fun problems. This review may help researchers to find appropriate symmetric games to capture the characteristics of real world strategic problems.

**Keyword:** Game Theory, Symmetric Games, Social Dilemmas, Prisoner's Dilemma, Chicken, Stag-Hunt, Deadlock.

## 1 Introduction

The main nature of a game is the interactive decision making in strategic situation in which at least two participants are involved and each player's decision depends on other players' actions. Each game consists of three main elements as follows:

- *Players* are set of decision makers.
- *Strategy* refers to the player's choices.
- *Payoff* is the specified value for each player in every combination of strategies.

Game theory deals with the studying of games. Decision makers do not have the same behavior in different situations, because the game's structure is not the same. Games are classified from some aspects as follows: Zero-sum game, symmetric game, coordination game, sequential, simultaneous game, etc.

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We concentrate on symmetric games including Prisoner's dilemma, Chicken, Stag-Hunt and Deadlock, The first three games are also known as two-person social dilemmas. These games have various applications for real world strategic situations.

Kollock [1] reviewed social dilemma in two-person and N-person categories and discussed possible solutions for them. Weber *et al.* [2] classified effective factors in decision making process in social dilemmas with experimental literature. In more recent research, Heus [3] answered to this question that if valence effects have any role in cooperation or defection. In the Chicken game, they found more defection in the loss-framed structures (negative outcomes) than in the gain-framed structures (positive outcome), but there is no difference between these two frames in Prisoner's Dilemma. Kusakawa *et al.* [4] studied the effect of a third person in a one-shot Prisoner's Dilemma. In this experimental study, the third person observes two players' game first and then plays with them later; in this case because of the observation of third person the mutual cooperation rate is improved. Ahn [5] stated the motivation of players to defection in prisoner's Dilemma.

Jurisc [6] reviewed strategies in iterated prisoner's Dilemma. Chong [7] introduced a different payoff structure for iterated prisoner's Dilemma in which payoff is not fixed for each strategy. Rezaei [8] studied evolution of cooperation by a social network model in the N-player version of Prisoner's Dilemma. Pathak [9] reviewed various strategies in repeated prisoner's Dilemma. He concluded cooperation is possible in repeated version, unlike the one-shot prisoner's Dilemma.

Applications of these games are in many areas; in economy [10, 11], Politics [12, 13], Water resources management [14], Network [15, 16], etc.

The first part of this review introduces some of the classical games and their characteristics. In the second part, we only focus on a group of them; symmetric games (Our motives and reason for this selection be explained). They are studied from different perspectives: history, structure, behavior of players, outcome and equilibrium. Moreover, main applications of them are reviewed. Simultaneously studying symmetric games and their applications is very beneficial. It provides accurate insight about games structure and helps us model and analyze real-world problems by symmetric games.

This paper is organized as follows: In section 2, we introduce sixteen common games briefly. Symmetric games are studied in section 3. In each game, history of game, scenario, the most likely outcomes, structure, equilibrium and applications of games are studied. Section 4 provides a comparison between four game's situations and finally conclusions are discussed in section 5.

## 2 Classification of Games

Games have different aspects. So, they can be classified from different perspectives. We will state some of these categories [17]:

- Players will have to make decision without knowledge of what others do. It means, Players' decision is made simultaneously. This kind of games are called simultaneous games.
- Zero-sum games capture situations that a player's gains is equal to another player's loss. A zero sum game is a special case of a constant sum game in which the sum of players' outcomes in each state is equal to zero.
- Another classification of games concerns the players' strategy. So games are divided into two groups; symmetric and *asymmetric games*. In symmetric games, player's evaluations

of a strategy depend only on the other players' strategy and it is independent of who is playing against them, i.e. "the player does not change the relative order of the resulting payoffs facing that player".

- A game in which the players benefit from mutual working in same direct is a *coordination game*. By contrast, a game is said to be *anti-coordination* in which players have the best payoff when they choose opposite strategy.

List of our selected games is given in table 1. They have simple structure, but are instructive and have included interesting scenarios. We only summarize their characteristics (cf the References for further studies of each game).

Symmetric games in table 1 are four games; Prisoner's Dilemma, Chicken, Stag-Hunt and Deadlock. Our motivation of studying symmetric games is as follows:

- They are built very well and are displayed in 2\*2 matrixes.
- They have simple and generic structures that recur in many diverse situations. Symmetric games are highly applicable. For example a situation occurs in different areas from political range to biology, but all of them have the same structural features. Other games are not much general and they are dependent on their own scenario.
- Transferability of these structures between different areas gives the study of symmetric game its importance in the game theory.
- These games are most commonly studied. There are numerous papers about them, and many
- researchers have used them to real-w

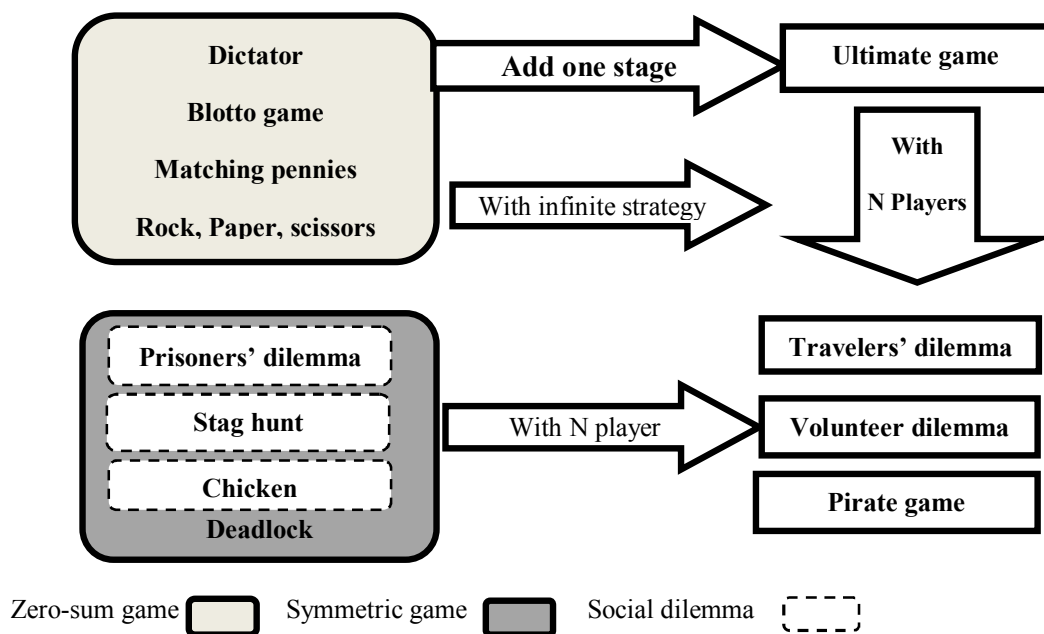
**Table 1** Characteristics and types of common games

| Games |                                | Characteristics of game |                                 |                                 | Types of game |          |              |              |
|-------|--------------------------------|-------------------------|---------------------------------|---------------------------------|---------------|----------|--------------|--------------|
|       |                                | Number of players       | Number of strategies per player | Number of pure Nash equilibrium | symmetric     | Zero-sum | Coordination | Simultaneous |
| 1     | Chicken game <sup>1</sup> [18] | 2                       | 2                               | 2                               | *             |          |              | *            |
| 2     | Prisoner's Dilemma[19]         | 2                       | 2                               | 1                               | *             |          |              | *            |
| 3     | Stag hunt <sup>2</sup> [20]    | 2                       | 2                               | 2                               | *             |          | *            | *            |
| 4     | Deadlock[18]                   | 2                       | 2                               | 1                               | *             |          |              | *            |
| 5     | Matching pennies[17]           | 2                       | 2                               | 0                               |               | *        |              | *            |
| 6     | Battle of sexes[21]            | 2                       | 2                               | 2                               |               |          | *            | *            |
| 7     | War of attrition[22]           | 2                       | 2                               | 0                               |               |          |              |              |
| 8     | Trust game[23]                 | 2                       | Infinite                        | 1                               |               |          |              |              |
| 9     | Ultimatum[24]                  | 2                       | Infinite                        | Infinite                        |               |          |              |              |
| 10    | Dictator game [25]             | 2                       | Infinite                        | 1                               |               | *        |              |              |
| 11    | Volunteer's Dilemma[26]        | N                       | 2                               | 2                               |               |          |              | *            |
| 12    | Pirates game[27]               | N                       | Infinite                        | Infinite                        |               |          |              |              |
| 13    | Blotto[28]                     | 2                       | variable                        | Variable                        |               | *        |              | *            |
| 14    | El farol bar[29]               | N                       | 2                               | Variable                        |               |          |              | *            |
| 15    | Rock, Paper, Scissors          | 2                       | 3                               | 0                               |               | *        |              | *            |
| 16    | Traveler's dilemma[30]         | 2                       | N                               | 1                               |               |          |              | *            |

<sup>1</sup> This game also known as hawk-dove or snow-drift. The structure of these games is the same, but the scenarios are different.

<sup>2</sup> Stag hunt have two other names: assurance game and trust dilemma.

Relationships exist between some of these games. By changing an element of a game, another game might be constructed. These relationships are depicted in figure 1.



**Fig. 1** Relationship between common games

Prisoner's Dilemma, Chicken and Stag-Hunt are also known as two-person social Dilemmas. Social Dilemmas are situations in which the individual interest is in contrast with public interest [31]. They are situations in which "individually reasonable behavior leads to a situation in which everyone is worse off than they might have been otherwise"[1].

This term is introduced by Dawez [32] to describe specific situations in which:

- I. Each player has a dominant strategy, i.e. the best payoff for him regardless of the strategy of the other players.
- II. If players choose their dominant strategy, then collective results are less preferred by all players than the result which would have occurred if all had not chosen their dominating strategy.

Liebrand [33] in 1983 relaxed the first requirement and defined a Social dilemma as a situation in which: each player has a strategy that yields the player the best payoff in at least one configuration of strategy choices, and that has a negative impact on the interests of the other players. The choice of that particular strategy by all persons results in a deficient outcome.

Social Dilemmas are mostly studied in literature of sociology, psychology, politics and economics. Kollock[1] reviewed literature of social dilemmas in two directions: first, kinds of social dilemmas included two-person Dilemmas (Prisoners' Dilemma, game of Chicken and Stag-Hunt) and multiple-person dilemmas (Public good dilemma and Common dilemma). Second, he considered possible solutions to social dilemmas. Weber[2] reviewed decision making process in social dilemma with experimentally focused literatures. They classified effective factors in decision making in two groups; individual differences (motivation, gender, personality and other) and situational factors (task structure and perceptual factors). Kopelman[34] specially reviewed factors influencing cooperation in commons dilemmas. (For more study of social dilemmas see Van Lange & Messick[35] and Kollock[1])

Now we focus on symmetric games from table 1; Prisoner's dilemma, Chicken, Stag-Hunt and Deadlock. Literature in the symmetric games can be classified into four groups:

**Table 2** Categories of symmetric games' literature

| Num | groups                                                                                 | Example                |
|-----|----------------------------------------------------------------------------------------|------------------------|
| 1.  | Games' structure, players' motive and factors affecting result of games.               | [5], [4], [3], [36]    |
| 2.  | Applications of games and how they relate to real-world situations.                    | [11], [14], [37], [38] |
| 3.  | Solve the game with variant solution concepts and discussion about equilibrium states. | [39]                   |
| 4.  | N-players and iterated version of games (discussion about variant strategy).           | [8], [9], [6]          |

In this study, we review first- two groups; description and applications of symmetric games. We believe studying symmetric games and their applications is important because it:

- States that they are not just interesting or fun problems.
- Provides accurate insight about the game's structure.
- Reveals differences between these games
- Represents how real-world problems can be modeled by them.
- Helps in classifying and analyzing real strategic situations

### 3 Symmetric games

A game is said to be symmetric if the following two conditions are satisfied[17]:

- Players' set of strategy is the same.
- Players' payoffs do not change by displacing their strategies. In a 2\*2 symmetric game, this condition means<sup>1</sup>:

$$U_1(S_1, S_2) = U_2(S_2, S_1) \text{ for every pair of strategy } (S_1, S_2)$$

By contrast, an asymmetric game is any game that violates at least one of the above requirements.

The overall strategy in the symmetric two-players game is based on two kinds[17]; cooperation and Absence of cooperation (defection). The behavior of the players in these games is as follows: Altruistic, Cooperative, Competitive, Individualistic and Free riding[40], [41]. If we consider a symmetric game, players' choices lead to four results [42]:

1. Both players choose cooperation strategies; this situation is called mutual cooperation (CC). Here collective benefit is maximized and parties are rewarded for this cooperation. Each one receives a payoff "R".
2. Both players do not cooperate (defection strategy). This situation is called mutual defection (DD) and they are punished for their choice; this result is shown by "P" which is the worst collective result.
3. Finally, if one player cooperates and the other does not, the Cooperator receives the lowest payoff "S" and the defector receives the largest payoff "R".

These results are summarized in figure 2.

|          |                   | Player 2           |                        |
|----------|-------------------|--------------------|------------------------|
|          |                   | Cooperation        | Absence of cooperation |
| Player 1 | Cooperation       | Reward, Reward     | Sucker, Temptation     |
|          | Don't cooperation | Temptation, Sucker | Punishment, Punishment |

**Fig. 2** Payoff matrix for symmetric 2\*2 games

<sup>1</sup>  $U_i$  represents player i's payoff function

We study each game from several perspectives: a brief history, scenario, structure, outcomes, equilibrium and applications of the game.

### 3.1 Prisoner's Dilemma

Prisoner's dilemma was originally framed by Merrill Flood and Melvin Dresher while working on game theory at RAND<sup>1</sup> Corporation in 1950. Then, Albert W. Tucker<sup>2</sup> at the school of Psychology in Stanford formalized the game with prison sentence payoffs and gave it this name[43].

This game was designed to represent a conflict of interests in the form of individual and group decision-making. On the other hand, this game shows Nash equilibrium solution does not always fulfill satisfaction of the players[18].

#### 3.1.1 Scenario

There are different versions of this game. Here, we explain the most common version[33]:

Two suspects are arrested, but the police do not have enough information for a conviction. Police incarcerate two persons in two separate cells, so that there is no connection between them. Two prisoners are faced with two options; remain silent or confess to crime. If both remain silent, both go free after a short time due to lack of evidence for conviction. If either of two prisoners confesses to crime and the other one remains silent, the silent prisoner is sentenced to five years while another goes free. If both persons confess, through their cooperation with police, they are sentenced to three year.

#### 3.1.2 Outcomes

Players' payoff can be shown by negative numbers that represent years of imprisonment. Since each player's payoff is negative, results are rating as positive values for each state in figure 3 [18].

In this situation, while one player gains his best payoff (three), the other one will be faced with his worst (zero). It means confess on opponent's silence (free riding behavior) provides the highest utility.

|         |            | Player2    |         |
|---------|------------|------------|---------|
|         |            | stay quiet | Confess |
| Player1 | stay quiet | 2,2        | 0,3     |
|         | Confess    | 3,0        | 1,1     |

Fig. 3 Payoff matrix of Prisoner's dilemma

#### 3.1.3 Structure

Structure of prisoner's dilemma -regardless of the scenario- can be considered as follows:[18]

Result of CC is better than DD, the differences of these two outcomes equal to  $(R - P)$ . It's the reward of cooperation. There should be an incentive for players to defect, so utility of DC is considered higher than CC. Meanwhile, both players are afraid of state DC and facing to the worst outcome.

To form a prisoner's dilemma situation the following Preference ordering is required[18]:

$$T > R > P > S, \quad R > (T + S)/2$$

<sup>1</sup> RAND is based on Research and Development.

<sup>2</sup>He was the chair of Princeton's mathematics department in the 1950s and 1960s.

What has made this game a puzzle that always for each player defection is better than cooperation regardless of the opponent's decision? In other words, each player has a dominant strategy. But when both decide to choose the dominant strategy, the result is not the best [18].

Lave [36] described three factors which affect cooperation in Prisoner's Dilemma; The number of trials the game will be repeated, relative value to each other's payoff and the characteristics of other player.

The motivation of player to defection is two factors: fear and greed. It means persons do not cooperate due to following reasons:

- Defection due to fear of opponent's defection that player is faced with the worst outcome. Here, Potential cost of cooperation is equal to payoff difference between punishment and sucker ( $P - S$ ). Defection based on fear when change to cooperation that players are sure cooperation of each other.
- Another motivation of defection is greed. Players tend to respond opponent's cooperation with defection. Here, person will benefits equal to payoff difference between reward and temptation ( $T - R$ ). Because of greed motive any commitment before the game does not lead to joint cooperation. [5], [44]

Moral tragedy occur since both players are willing to cooperate but do not cooperate, because they do not have trust to each other. They defect just think to other defect. In other words, they are misunderstood about the intentions of each other. William Poundstone states about the first person confess in criminal gangs [18]: "Each one of a gang, so placed, is not so much greedy of reward, or anxious for escape, as fearful of betrayal, he betrays eagerly and early that he may not himself be betrayed".

### 3.1.4 Equilibrium

In this game, each player has a dominant strategy ( $3 > 2$  and  $1 > 0$ ). It means:

$$U_1(D, S_2) > U_1(C, S_2) \quad \text{and} \quad U_2(S_1, D) > U_2(S_1, C)$$

Mutual defection (DD) is Nash equilibrium, because players do not have incentive to move. Mutual cooperation (CC) is Pareto optimal, where group utilities maximize. Here Pareto optimal solution is not equal to Nash equilibrium.

### 3.1.5 Applications

Prisoner's dilemma is not merely a fun game. Many situations and human interactions have the similar structures. "Prisoner's Dilemma is important not because we are interested in understanding the incentives for prisoners to confess, but because many other situations have similar structures." [17]

Many applications of this game in various fields are proof of this claim. Here we describe one of them in detail [45] and summarize some other applications in table 4.

Alliance between two companies can be considered as a critical decision for the growth and profitability of them. This is mutual decision problem which is dependent on two firms.

Each firm has two choices: Sharing the high level or low level of firm's resources. Resources include human resources, organizational knowledge, equipment, assets, time, credit and capital. There are costs associated with the first option. The most important of them is opportunity cost. In general, firm can spend its resources to other tasks for example entering to new market. On the other hand, choosing the first option leads to a mutual alliance which creates a benefit to both firms. Some of these benefits reduce the cost of advertising and marketing, reduce the production's cost per unit, mutual learning and access to joint resources and equipment. In summary, they have reached a high level of synergy. Each firm is aware of the alliance' benefits, but only they cooperate when the other firm is prepared to cooperate. Otherwise, the alliance revenue is less than its cost.

These conditions of the problem can be considered a prisoner's dilemma situation. In which two firms (players) involved in the decision making. Their decisions are interdependent and player's gain

depends on the strategy of another firm. Each firm has two strategies; cooperative and non-cooperative. Cooperative behavior means sharing the high level of firm's resources and non-cooperative behavior implies sharing the low level of resources. Two firms have to decide simultaneously. Furthermore, the decision is irreversible, it means that if they decide to cooperate or do not cooperate, they cannot change their decision due to contractual and legal restrictions. The modeling of this situation is as follows:

Each firm can share a high level of its resources. The cost of this choice is  $I_c$ . It can also share a lower level of resources with the cost  $I_d$  the  $I_c < I_d$  obviously. Non-cooperation is a Personal decision that leads to cost saving, but its result will affect other company's gain. Mutual cooperation ( $P_{cc}$ ) provides high utility for group. Mutual defection ( $P_{dd}$ ) has minimum group utility. Payoff function of firms is displayed on figure 5.

| Mutual cooperation     | Mutual defection       | Temptation(defection on Cooperation's cooperation) | Sucker(cooperation on Defection's cooperation) |
|------------------------|------------------------|----------------------------------------------------|------------------------------------------------|
| $c = (P_{cc}/2) - i_c$ | $d = (P_{dd}/2) - i_d$ | $w = (P_{cd}/2) - i_d$                             | $s = (P_{cd}/2) - i_c$                         |

Fig. 4 Payoff function of alliance game[45]

So that  $w > c > d > s$  and  $2c < w + s$ . also  $c > 0$  (o implies opportunity cost of firm).

|        |                 | Firm 2    |                 |
|--------|-----------------|-----------|-----------------|
|        |                 | cooperate | Don't cooperate |
| Firm 2 | cooperate       | $c, c$    | $s, w$          |
|        | Don't cooperate | $w, s$    | $d, d$          |

Fig. 5. Payoff function of alliance game[45]

In this problem -similar to original prisoner's dilemma- every player has a dominant strategy. That is defection, it means for each firm defection is better than cooperation. At the same time there is incentives of betray, players are encouraged to think about cooperation because of interdependent decisions and the potential benefits of alliance. Each player incentive to maximize its profit (output alliance) by minimizing its costs (input alliance).

### 3.2 Chicken Game

The game of Chicken has very interesting history as follows:

"*Rebel without a Cause*" is a 1955 American drama film directed by Nicholas Ray. This film is about abnormalities of the young Americans after world wars. Two men drive toward a cliff. The winner is who does not stop and the first driver to stop loses. Winner shouts "*Chicken*" and to humiliate looser man[18].

Another scenario is the story of *Russell* in his book. He wrote "*Common sense and nuclear weapons*" in 1959 and describes a situation that two countries are reluctant to build nuclear weapons but they compete with other just due to pride. He compared the nuclear standoff and the cold war with the Chicken game[18].

#### 3.2.1 Scenario

Suppose that two drivers speed towards each other (in film, they drive toward cliff). The first to swerve loses and Chickens out.[17]Each player tends to the other swerve first. As they get close to

each other, mutual destruction becomes more and more imminent. Disaster occurs when none of players will not swerve and continue straight crazy. The first to swerve is considered to be “Chicken” and becomes an object of contempt[18].

### 3.2.2 Outcomes

All of the possible results are shown in figure 2 [18]. Outcomes are given by the payoff matrix in fig 6.

|         |          | Player 2 |          |
|---------|----------|----------|----------|
|         |          | Swerve   | Straight |
| Player1 | Swerve   | 2,2      | 1,3      |
|         | Straight | 3,1      | 0,0      |

Fig. 6 Payoff matrix of Chicken game

### 3.2.3. Structure

The structure of Chicken game -Regardless of the scenario- can be described as follows:

No player has a dominant strategy. Each player wants to do the opposite of what the other player does. It means player one decides to swerve if he or she is certain that player two drive straight.

Punishment's outcome is disastrous to all the players (for example: war in military applications or bankruptcy in economic applications of Chicken). Both players have the following preference ordering:

$$T > R > S > P$$

It is clear Chicken is a dangerous game. The only way to win this game is defection, while this strategy is associated with high risk[46].

### 3.2.4 Equilibrium

There are two pure Nash equilibriums in the game of Chicken, in which one player swerves and the other drives straight<sup>1</sup>. In this game, anti-coordination strategies create Nash equilibrium. Cooperate on opponent's defection and defection on opponent's cooperation is the Nash equilibrium state. R, T and S are Pareto optimal. Group's utility is maximized in these states.

### 3.2.5 Applications

Chicken is appeared in conflicts and clashes between the two sides. There are many applications in the literature. Here, we describe an application of Chicken in economics[37] and others are summarized in table 4.

During the economic crisis, governments adopt protectionism<sup>2</sup> to led demand-side to domestic production. The purpose of these restrictions is protectionism among foreign producers. Economists always give hints about the consequences of these policies. They warn that the adoption of this policy may take retaliatory response, what reduce to the demand for domestic products. And by the way, the

<sup>1</sup>There is also a mixed-strategy equilibrium for the game of Chicken

<sup>2</sup> Governments restrict international trade to protecting local businesses and jobs from foreign competition. Typical methods of protectionism are import tariffs, quotas, subsidies or tax cuts to local businesses and direct state intervention.

crisis is worse. For example, when the United States increased 60% import's tariff over more 3000 products in 1929, at least 60 countries imposed tariffs in response to America's action. This situation lead to a reduction of 70% in world trade.

In general, country A could adopt policies that support domestic production, it has a negative impact on exports of country B. country B can also ban imports from country A in response to A's action(it reduces its export Consequently).

Therefore, the countries' possible strategies are to continue the status quo or trade barrier. Here we suppose that result of a trade war (both country choose trade barrier) is worse than every other state for each player. The assumption is close to reality.

The structure of this decision problem is based on Chicken game. The best state for country A is trade barrier (stop importing from country B) while country B choose status quo (to continue its exports to that country B). Disaster occurs when both countries also decide to trade barrier, where both countries' gain reduces because of reduced export.

|           |               | Country B  |               |
|-----------|---------------|------------|---------------|
|           |               | status quo | trade barrier |
| Country A | status quo    | 0,0        | -2,1          |
|           | trade barrier | 1, -2      | -3, -3        |

Fig. 7 Payoff matrix in protectionism game[37]

Boffaand and Olarreaga[37] expressed that country A adopts on protecting policy against countries that are less likely to retaliate (GDP differences between countries is the measure). They said: “you do not mess with the lion, you go after the sheep”

### 3.3. Stag-Hunt

The Stag-Hunt is a story that became a game. The game is a prototype of the social contract [20]. In this game, players choose between strategically safe (defection) and risky actions (cooperation). Stag-Hunt is in class of coordination games that have multiple Nash equilibrium. Other names for this game are "assurance game" and "trust dilemma"[18].

#### 3.3.1 Scenario

Stag-Hunt was introduced by Jean Jacques Rousseau in “Discourse on the origin and foundations of inequality among men” this way:

Two hunters can either jointly hunt a stag or individually hunt a rabbit. Hunting stags is quite challenging and requires mutual cooperation. If either hunts a stag alone, the chance of success is minimal. There is no communication between the two hunters. The Stag yields more food than the Rabbit. Both hunters are rational and equally informed[18].

There are two important points in this story[20]:

- What are the values of a rabbit and of an individual's share of the Stag given a successful hunt?
- What is the probability that the hunt will be successful if all participant faithful to the hunt?

David Hume[18] also has another version of this game. His most famous illustration of a convention has the structure of a two person Stag-Hunt game. Two men who pull at the oars of a boat, do it by an agreement or convention, they have never given promises to each other. Both men can either row or not row. If both row, they get the outcome that is best for each man just as in Rousseau's example, when both hunt the stag. If one decides not to row then it makes no difference if the other does or not - they do not get anywhere. The worst outcome for first man is he row and the other does not[18].

### 3.3.2 Outcomes

Matrix form of game is displayed in figure 8.

|         |        | Hunter 2 |        |
|---------|--------|----------|--------|
|         |        | Stag     | Rabbit |
| Hunter1 | Stag   | 3,3      | 1,2    |
|         | Rabbit | 2,1      | 2,2    |

**Fig. 8** Payoff matrix of Stag-Hunt

### 3.3.3 Structure

In this game, players do not have dominant strategy. If two players cooperate, then the best result is obtained. It means that this outcome is best for both players even from the states that one of the players tempts. The two players' preference ordering is as follows[18]:

$$R > T \geq P > S$$

Cooperation strategy in the Stag-Hunt is the best strategy and provides high utility for both players. But players do not cooperate always. Here the only defection motive is fear and there is no motive of greed, unlike prisoner's dilemma. Both players prefer stag to rabbit, but rabbit also is better than nothing. It means hunting stag is most beneficial, but requires a lot of trust among players. If both players have to trust each other, they choose cooperation and have maximum utility[18].

Free riding does not mean in Stag-Hunt. First player cooperate if he knows second player cooperate too. If players trust each other, there is no risk of noncompliance. Defection is a risk free strategy always.

### 3.3.4 Equilibrium

A Stag-Hunt game has two pure Nash equilibria; an efficient equilibrium (R) and an inefficient one (P). R is equilibrium of mutual cooperation, is called Pareto efficient equilibrium (payoff dominant). P is equilibrium of mutual defection, is called Pareto inefficient equilibrium (risk dominant). Players in these states do not have any incentive to move, where both have access to hunting. there is high-payoff equilibrium in which both hunt stag and a low-payoff equilibrium in which both hunt rabbit[48].

Uncertainty about the other player's action may prevent them to take such strategic risk. The interpretation of the equilibrium points that both hunters prefer stag to rabbit but rabbit to nothing [18].

### 3.3.5 Applications

The Stag-Hunt game provides a certain situation that decisions of the players are based on trust each other. Kollock [1] believes although the Stag-Hunt has attained less attention than the prisoner's dilemma, it provides a more accurate explanation of the real life situations.

Here we describe an application of Stag-Hunt in water resources management [14]. Two littoral countries are going to share a lake. Each country has a river that flows into the lake. The water level is reducing in some seasons, due to water evaporation. In this season, two country most increase water release to the lake. If just one country increases flow, the situation does not improve. If both countries release their water into the lake, the environmental benefit would be more

than lost revenue from decreasing upstream consumptive use. The two countries face the situation shown in figure 9.

|         |                | Country 2 |                |
|---------|----------------|-----------|----------------|
|         |                | Increase  | Don't increase |
| County1 | Increase       | 3,3       | 1,2            |
|         | Don't increase | 2,1       | 2,2            |

**Fig. 9** payoff matrix of common environmental game[14]

### 3.4 Deadlock

The last symmetric game we discuss is Deadlock. Similar to the Prisoner's Dilemma each player does better defecting regardless what other player does, but unlike Prisoner's dilemma the result of punishment is better than reward for both players[18]. It means result of mutual defection is better than mutual cooperation and any outcome. We could assign a payoff matrix to this game as figure 10.

|         |                 | Player2   |                 |
|---------|-----------------|-----------|-----------------|
|         |                 | Cooperate | Don't cooperate |
| Player1 | Cooperate       | 1,1       | 0,3             |
|         | Don't cooperate | 3,0       | 2,2             |

**Fig.10** Payoff matrix of Deadlock

This game has one Nash equilibrium in which both player defect. Pareto equilibrium is equal to Nash equilibrium, i.e. individual rationality leads to collective rationality. Preferences of the different result are as follows:

$$T > P > R > S$$

This structure makes Deadlock less interest and challengeable than other symmetric games. According to Poundstone [18] "Deadlock is the least troublesome of the symmetric games".

## 4 Comparison

In each game, there are two players. Every player has two strategies; cooperation and defection. Players make decision simultaneously. In prisoner's dilemma every player has a dominant strategy. The best outcome is temptation and sucker's outcome has the lowest payoff. If two players defect, the player's utility is lower than mutual cooperation, but is not the worst. Chicken game, is a type of prisoner's dilemma in which utility of sucker and punishment is changed [18]. It means that sucker is preferred to punishment. Mutual defection (DD) leads to punishment's outcome. In Chicken, P is disaster, but in prisoner's dilemma P only has fewer utility. In Chicken, the worst outcome occurs if both players defect whereas the worst outcome in Prisoner's Dilemma is that in single cooperation. Stag-Hunt is a prisoner's dilemma that utility of reward and temptation is changed [18]. In other words, mutual cooperation provides best result, even better than single defection.

In Stag-Hunt, if players trust together then there is no incentive to defection (there is not greed). But in prisoner's dilemma, motivations for defection never disappear due to the fear or greed of the players[5]. Characteristics of these four games are displayed in table4.

**Table 3** Comparison of symmetric games

| symmetric games    | Type of game |                   | Preferences        | Dominant strategy | Equilibrium |         |
|--------------------|--------------|-------------------|--------------------|-------------------|-------------|---------|
|                    | Coordination | Anti-coordination |                    |                   | Nash        | Pareto  |
| Prisoner's dilemma | *            |                   | $T > R > P > S$    | Defection         | P           | R       |
| Chicken            |              | *                 | $T > R > S > P$    | -                 | T, S        | R, S, T |
| Stag-Hunt          | *            |                   | $R > T \geq P > S$ | -                 | R, P        | R       |
| Deadlock           | *            |                   | $T > P > R > S$    | cooperation       | R           | R       |

**Table 4** Selected applications of symmetric game in literature

| Type of game |    |    | Field                      | Objective(s)                                                                                                                                                            | Citation |
|--------------|----|----|----------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------|
| PD           | CH | SH |                            |                                                                                                                                                                         |          |
| *            |    |    | Economics                  | analyze the intraday behavior of banks with respect to settlement of interbank claims                                                                                   | [49]     |
| *            |    |    | Economic                   | choose uniform pricing or discriminatory pricing policy                                                                                                                 | [11]     |
| *            |    |    | Economic                   | modeling the profit maximizing behavior of bankers and the investors                                                                                                    | [38]     |
| *            | *  | *  | Water resources management | applications of game theory to water resources management through a series of non-cooperative game and illustration of the dynamic structure of water resource problems | [14]     |
| *            | *  | *  | Water resources management | Applications of game theory for water resource conflict                                                                                                                 | [50]     |
| *            |    |    | Management                 | the effect of endogenous matching on peoples' willingness to cooperate                                                                                                  | [51]     |
| *            |    |    | Economic                   | modeling firms' optimal strategy under a price war with market demand growth using repeated Prisoner's Dilemma                                                          | [52]     |
| *            |    |    | Network                    | the cooperative behaviors in an online friendship network                                                                                                               | [53]     |
| *            |    |    | Social                     | cooperation in iterated prisoner's dilemma game in a drug-dependent population                                                                                          | [54]     |
| *            |    |    | Network                    | the evolutionary Prisoner's Dilemma game on community network                                                                                                           | [16]     |
| *            | *  |    | Politics                   | the possibilities and limitations of game theory as an instrument for the study of international relations                                                              | [12]     |
| *            | *  |    | Politic                    | Applications of "Prisoner's Dilemma" and "Chicken" Models in International Politics.                                                                                    | [13]     |
|              | *  |    | Network                    | Discussion about strategy evolution in the game of Chicken using network                                                                                                | [55]     |
| *            |    |    | Economic                   | price and quantity competition of firms in undifferentiated products                                                                                                    | [56]     |
| *            | *  |    | International negotiations | Analysis international environmental negotiations                                                                                                                       | [57]     |
| *            |    |    | Routing                    | A review of game theoretical methodologies in routing models                                                                                                            | [15]     |
| *            |    |    | Management                 | modeling intergroup conflicts over continuous public goods                                                                                                              | [58]     |
| *            | *  | *  | Finance                    | use of real options valuation and game theory to analyze investment opportunities involving strategic decisions                                                         | [59]     |
| *            |    |    | Economic                   | state three situations in the contemporary economic; gold mining, the level of taxes in the two countries and competition of two countries for labor                    | [10]     |
| *            | *  |    | Economic                   | Descriptions of how game theory has been used to pricing model                                                                                                          | [60]     |

## 5 Conclusions

Studying structures of symmetric games helps us refine our intuition about the games and allows us to improve our ability to model the real-world situations in different areas. In this study some of the classical games are introduced. Then structures and applications of symmetric games including Prisoner's Dilemma, Chicken, Stag-Hunt and Deadlock are reviewed. We have considered two points; Structural description of different situations and applications of these games. We emphasize that these games can be observed everywhere in the real life.

Symmetric games have generic structures that are considered in many diverse situations. They are highly applicable, especially Prisoner's dilemma. In prisoner's dilemma every player has a dominant strategy. The best outcome is temptation and sucker's outcome has the lowest payoff. In chicken game, sucker is preferred to punishment. In stag hunt, mutual cooperation provides best result, even better than single defection

This paper provides the basic concepts of game theory regardless of complex quantitative aspects and indicates the essence of game theory by review symmetric games and variant applications of them. It also states implicitly that game theory deals with the problem exactly. Since game theory is an interdisciplinary field, this paper is the first step in understanding concepts of games and may help researchers how game theory is useful in their research problems.

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