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# Optimizing a Hierarchical Hub Covering Problem with Mandatory Dispersion of Central Hubs

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**Abstract** The hierarchical hub location problem is encountered three-level network that is applied in production-distribution system, education system, emergency medical services, telecommunication network, etc. This paper addresses the hierarchical hub covering problem with single assignment accounting for mandatory dispersion of central hubs restriction as a special case. This formulation with incorporating mandatory distance and covering constraints has not been remarked in the hierarchical hub location problem literature. To test the performance of the problem on the Turkish data, computational experiments carried out and the model outcomes give useful insight associating with the model sensitivity to its parameters.

**Keywords**: Hub location, Hierarchical hub network, Covering problem, Mandatory dispersion constraint.

## 1 Introduction

Hub location problem is a comprehensive and novel issue in facility location area and has many applications arise in transportation systems, telecommunication network, cargo delivery, product and distribution, supply chain management. Whereas direct connection of origin-destination nodes is impossible and needs high investment and operational costs therefore, in order to take advantage economical scale, flows (passengers, information, goods and etc) are consolidated at hubs in their routes. Hence flows at hub points are collected (origin-hub), consolidated (hub-hub) and distributed (hub-destination). The hub problem's objective is to minimize total transportation cost, total transportation time or total nodes distance, so discount factor α ($0 \le \alpha \le 1$) is used between hubs to account for economical scale.

O'kelly [1] that introduced the first mathematical formulation. Later on, O'kelly [2] developed the first quadratic mathematical formulation was minimizing the total cost of flows. The remaining of literature review included linearizing the mathematical formulation to the hub location problem. In this aspect, one recent paper by Alumur and Kara [3] studied and reviewed more than 100 papers. They classified hub location problem into four sub-problem including p-hub median problem, p-hub center, p-hub covering and hub location with

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fixed cost. Also Farahani et al [4] reviewed the hub location papers in all four sub-problems and finally, future trend research was given.

First, Campbell [5] introduced the hub covering problem. He provided three coverage criteria definitions are as follows:

- If the cost (time or distance) from node i to node j via hub k and hub m does not exceed a specific value
- If the cost (time or distance) for each link from node i to node j via hub k and m does not exceed a specific value
- If the cost (time or distance) for each link from origin-hub and hub-destination does not exceed a specific value

Also, he proposed p-hub center and hub covering model to the literature and provided more formulation for hub set covering problem as well as hub maximal covering problem for both single and multiple allocation in hub covering problem. Kara and Tansel [6] proposed nonlinear binary integer programming and the various linearizations for old and new formulation. They proved that the hub covering problem is Np-hard. Ernst et al [7] represented that mathematical formulation for hub covering problem with single assignment and reported their model was solved in less computational time compared to Kara and Tansel's model also first, they discussed using the cover "radiuses" idea for the hub covering problem.. A new formulation for single assignment in hub covering problem discussed by Wagner [8] that improved model formulation for hub covering problem.

First type of coverage was applied in the most researches, Karimi and Bashiri [9] proved that first coverage type is inefficient as the cost (time or distance) between origin and hub may be too large from other links. As present paper studies on the hub covering problem, the second type of coverage is considered.

The Most of papers in the literature of hub location problem studied two-level hub and spoke networks and three level hub and spoke network is called the hierarchical network that first studied by Yaman [10], has been less considered in the hub location problem. The hierarchical hub and spoke network, high level arises in connecting central hubs to each other in the complete network; the second level includes a star network connecting hubs to central hubs and in the third level, demand nodes connect to hubs and central hubs. Yaman [10] introduced the single assignment hierarchical network in p-hub median problem and enforced delivery time restriction to his model. Later some authors developed Yaman's paper on their work like as Davari and Fazel-Zarandi [11] considered network under uncertainty with fuzzy flows. Karimi et al [12] applied capacity constraint and Korani and Sahraeian [13] studied hub maximal covering problem in the hierarchical networks. Hence instead of focusing on the two-level hub and spoke network, this study discusses the hierarchical hub covering problem with mandatory dispersion of central hubs.

To improve high service level of central hubs to hubs and demand nodes by coverage constraints, in addition mandatory dispersion of central hubs constraint is employed in this work. In a hub location problem the failure risk exists in nodes. Mandatory distance $D_{man}$ for central hubs focuses on locating central hubs among N nodes as far away as possible therefore, crowded in central hubs does not impact the network. Originally, one may refer to dispersion distance issue is Kim and O'kelly [14] studied the mandatory distance of hubs in telecommunication network to keep hubs away from each other. Also Fazel-zarandi et al [15] proposed the hub covering problem with mandatory dispersion and back up coverage in their routes. Reviewing of papers published in the hub location problem area shows the mandatory dispersion problem is considered less interest to date. To the best of our knowledge, posing both covering and mandatory distance of central hubs restrictions have not been studied in the hierarchical hub location problem in the literature.

The outline of this paper is as follows: section two presents a mixed integer programming formulation for new hierarchical hub covering location problem with mandatory dispersion among central hubs. Numerical example, computational analysis and results for Turkish dataset appear in section three. Finally, conclusions and future research trend are remarked in section four.

## 2 Problem definition

In this section, the hierarchical hub location problem was addressed by Yaman [10] is considering the second type of coverage and mandatory dispersion of central hubs in order to increase service levels to demand nodes before giving the new mathematical formulation of the problem. Sets, parameters and variables are explained.

Let N is given as node set, $H \subseteq N$ be as possible location set for hub, $C \subseteq H$ be as possible location set for central hub. The number of hubs must be located in network is denoted by $P_H$ and the number of cenral hubs must be located in network is denoted by $P_C$, $\alpha_H$ and $\alpha_C$ be as discount factors between hub and central hubs and between two central hubs, respectively. $c_{ij}$ be as cost of transportation a unit flow for node $i \in N$ to $j \in H$ and is assumed that $c_{ij}=c_{ji} \geq 0$ for all $i, j \in N$, $c_{ii}=0$ for all $i \in N$.$u_{ijl}$ be amount of flow from $i \in N$ as origin or destination node traversing from hub $j \in H$ and central hub $l \in C$. $g_{ilk}$ be amount of flow from $i \in N$ as origin or destination node traversing from two central hub $l \in C$ and $k \in C$ as $l \in C \neq k$. coverage radius is determined by $r_H$ for hubs and $r_C$ for central hubs, respectively. $f_{ir}$ denotes the amount of flow must be traveled from node $i \in N$ to $r \in N$. $D_{man}$ be as mandatory distance between central hubs and limited to 0 and $D_{max}$ $(0 \leq D_{man} \leq D_{max})$ also M is the sufficient big number for mandatory dispersion constraint. $X_{ijl}$ is defined as binary variable if node $i \in N$ allocates to hub $j \in H$ and central hub $l \in C$ take a value 1 and 0 otherwise.

Now, the mathematical formulation of the hierarchical hub covering location problem with mandatory dispersion among central hubs is given as below:

$$\min \sum_{i \in N} \sum_{r \in N} (f_{ir} + f_{ri}) \sum_{j \in H} c_{ij} \sum_{l \in C} x_{ijl} + \sum_{i \in N} \sum_{j \in H} \sum_{l \in C, l \neq j} \alpha_H c_{jl} u_{ijl} + \sum_{i \in N} \sum_{l \in C} \sum_{k \in C, k \neq l} \alpha_C c_{lk} g_{ilk} \quad (1)$$

s.t:

$$\sum_{l \in C} \sum_{j \in H} x_{ijl} = 1 \qquad \forall i \in N \quad (2)$$

$$x_{ijl} \leq x_{jjl} \qquad \forall i \in N, j \in H, j \neq i, l \in C \quad (3)$$

$$\sum_{r \in N} x_{jrl} \leq x_{lll} \qquad \forall j \in H, l \in C, l \neq j \quad (4)$$

$$\sum_{j \in H} \sum_{l \in C} x_{jjl} = p_H \quad (5)$$

$$\sum_{l \in C} x_{lll} = p_C \quad (6)$$

$$\sum_{k\in C} g_{ilk} - \sum_{k\in C} g_{ikl} = \sum_{r\in N} f_{ir} \sum_{j\in H} (x_{ijl} - x_{rjl}) \qquad \forall\, i \in N, l \in C \tag{7}$$

$$\sum_{r\in N, r\neq j} (f_{ir} + f_{ri})(x_{ijl} - x_{rjl}) \leq u_{ijl} \qquad \forall\, i \in N, j \in H, l \in C, l \neq j \tag{8}$$

$$d_{lk} + M(1 - x_{lll}) + M(1 - x_{kkk}) \geq D_{man} \qquad \forall\, l \in C, k \in C, k \neq l \tag{9}$$

$$c_{ij}\, x_{ijl} \leq r_H \qquad \forall\, i \in N, j \in H, l \in C, l \neq j \tag{10}$$

$$c_{il}\, x_{ill} \leq r_C \qquad \forall\, i \in N, l \in C, l \neq i \tag{11}$$

$$\boldsymbol{\alpha}_C\, c_{jl}\, x_{jjl} \leq r_C \qquad \forall\, j \in H, l \in C, l \neq j \tag{12}$$

$$x_{ljl} = 0 \qquad \forall\, j \in H, l \in C, l \neq j \tag{13}$$

$$u_{ijl} \geq 0 \qquad \forall\, i \in N, j \in H, l \in C, l \neq j \tag{14}$$

$$g_{ilk} \geq 0 \qquad \forall\, i \in N, l \in C, k \in C, k \neq l \tag{15}$$

$$x_{ijl} \in \{0,1\} \qquad \forall\, i \in N, j \in H, l \in C \tag{16}$$

The objective function sums the transportation cost of flows overall, in the first term, the transportation cost of flow from node $i \in N$ to other node that traverses from hub $j \in H$ are calculated, the second term calculates the transportation cost of flow from $i \in N$ as origin or destination to other nodes which passes the connection routes of hub $j \in H$ and central hub $l \in C$ and the third term sums the transportation cost of flow from $i \in N$ as origin or destination to other nodes which passes the connection routes of two central hub $k \in C$ and $l \in C$ as $k \neq l$. since the present paper model considers the single allocation case, so that constraint (2) and (16) represent single allocation therefore, constraint (2) states every node is allocated to exactly one hub and one central hub. Constraint (3) expresses that a node i is allocated to hub j and central hub l, so node j should be a hub in the network and is assigned to central hub l. constraint (4) states that hub j cannot be assigned to another node unless that node be central hub. Due to constraints (5) and (6), the number of hubs and central hubs are determined. The main works of this paper are constraints (7) to (10). In order to improve service levels to demand nodes and preventing from network failure or disaster when emergencies condition accurse in the hierarchical hub network, Constrain (7) ensures that a mandatory dispersion exists between central hubs as there should be at least a distance of $D_{man}$ between them. For covering most of nodes, $r_H$ and $r_C$ state hubs and central hubs coverage radius, respectively through constraints (8), (9) and (10). Constraints (11) and (12) are the flow balance constraints. Constraint (13) is redundant but it strengthens the model. Constraints (14) and (15) calculate $u_{ijl}$ and $g_{ilk}$ values as flow variables and constraint (16) limit $x_{ijl}$ variable to take binary values.

## 3 Computational study

In this section, considering the well known Turkish dataset, numerical examples are explained and the efficiency of proposed model is discussed analyzing the influential parameters on cost function value, location of hubs and central hubs and computational time.

At first glance, one of the ways to deal with paper model is to solve proposed mixed integer programming model using GAMS 22.2 by commercial CPLEX 12.1 solver. Test problem was run on a 2 GHz, Intel ® Core ™ i5-2430M CPU, equipped with 4.00 GB of RAM.

### 3.1 Test problem

The well known Turkish dataset was introduced by Tan and Kara [16]. This dataset is employed for hub location area consistently. It includes flow. Times and distance between pair of nodes and also it provides fixed link cost and fixed hub cost. For our proposed model, we need flows and distance between nodes. In Turkish dataset, the main data is for 81 cities but in this paper, 34 cities that include important and populated cities are chosen. To assess some important and influential parameters of model, in the next sub-section, numerical examples are given and model results are analyzed based on total cost function, locating hubs and central hubs and run times changing some parameter value.

### 3.2 Analysis and results

Testing the mathematical formulation using the well known Turkish dataset is analyzed. In our computational design, in order to provide and evaluate model's behavior, the various values of model parameters on total cost function are given and the results are reported.

Turkey dataset with n=34 and pH=7 is used. The various values of discount factors ($\alpha_H$,$\alpha_C$) as $\alpha_H \geq \alpha_C$, coverage radius, number of central hubs and $D_{man}$ are considered. If $D_{man}$=0, the mandatory dispersion constraint in this model is removed and after that the comparison between model applying this restriction and without it reported. In addition, this dataset does not include coverage radius so it calculates in terms of transportation cost based on pair of nodes distance. The fact that, in this work, the second type of coverage (cost) is defined as constraints. The cost of pair of nodes is calculated by multiplying a fixed number cost in a unit at pair of nodes distance, the remarkable points is where no feasible solution in paper model is reported as infeasible. The interesting analysis should be carried out is the effect of $D_{man}$ and coverage radius on model output and the optimal objective function.

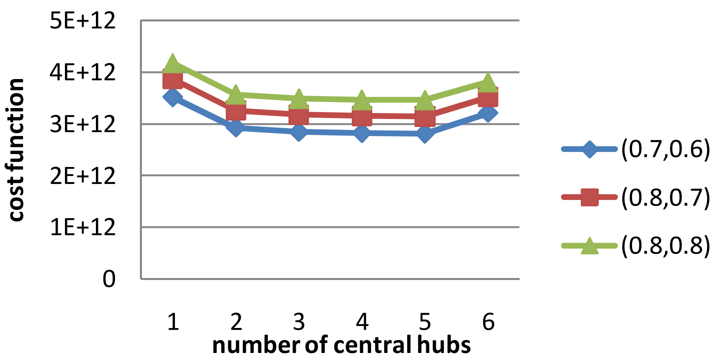


**Fig.1** The effect of discount factors on the cost function value

According to the results are reported for Turkish dataset, the increasing ($\alpha_H$,$\alpha_C$) from (0.7,0.6) to (0.8,0.7) and (0.8,0.8) with the different values $P_C$ leads to significance increasing on the values of cost function, which are depicted in figure .1, we conclude that discount value in cost function between hubs and central hubs and between central hubs is decreased.

The remarkable point for the hierarchical network, when the number of central hub is one, the network encounters with star network but the network results in the increase of number of central hubs that is equivalent with the number of hubs, the hierarchical network is changed into two-level network, all hubs are central hubs and demand nodes are allocated to them. For $(\alpha_H,\alpha_C)$ =(0.7,0.6) and $(\alpha_H,\alpha_C)$=(0.8,0.7) in table 1 and table 2.

**Table 1** objective function values of proposed model for $(\alpha_H,\alpha_c)$=(0.7,0.6), n=34 and $p_H$=7

| $(r_H,r_C)$ | | $D_{man}$ | | | |
|---|---|---|---|---|---|
| (46000,75000) | $P_C$ | 0 | 450 | 700 | 800 |
| | | OFV | OFV | OFV | OFV |
| | 1 | 3517897660499.999 | 3517897660499.999 | 3517897660499.999 | 3517897660499.999 |
| | 2 | 2926673990475.001 | 2926673990475.005 | 2926673990475 | 2926673990475.000 |
| | 3 | 2733975757500 | 2733975757500.000 | 2842176254874.999 | 2842176254875 |
| | 4 | 2672223674524.999 | 2672223674525 | 2820670838574.999 | 2853070673450 |
| | 5 | 2638211957175 | 2638211957075 | 2813568853500 | 3232808017624.999 |
| | 6 | 2616108920825 | 2620068782600.000 | 3211305145050 | Infeasible |
| | 7 | 2590766978900 | 2610784481149.999 | Infeasible | Infeasible |
| (85000,195000) | 1 | 2985061748900 | 2985061748900 | 2985061748900 | 2985061748900 |
| | 2 | 2749716774824.999 | 2749716774825.000 | 2852501835224.999 | 2852501835225.001 |
| | 3 | 2682873680125.000 | 2682873680125 | 2830901229824.998 | 283244817325.0005 |
| | 4 | 2645505201350 | 2647442811625 | 2814661848650.001 | 2853070673450 |
| | 5 | 2615765770775.001 | 2618603511475.000 | 2813568853500 | 3232808017625 |
| | 6 | 2595956577024.9976 | 2609507940400 | 3211305145050 | Infeasible |
| | 7 | 2574174196150 | 2610784481149.999 | Infeasible | Infeasible |

**Table 2** objective function values of proposed model for $(\alpha_H,\alpha_c)$=(0.8,0.7), n=34 and $p_H$=7

| $(r_H,r_C)$ | | $D_{man}$ | | | |
|---|---|---|---|---|---|
| (46000,75000) | $P_C$ | 0 | 450 | 700 | 800 |
| | | OFV | OFV | OFV | OFV |
| | 1 | 3864986590625.000 | 3864986590625.000 | 3864986590625.000 | 3864986590625.000 |
| | 2 | 3256978868300 | 3256978868299.999 | 3256978868300.000 | 3256978868300 |
| | 3 | 3053161932775 | 3053161932774.995 | 3180136530024.999 | 3180136530025.000 |
| | 4 | 3000428151250 | 30004281512250 | 3154849593825 | 3182687786700 |
| | 5 | 2966397596150.000 | 2966397596150 | 3144164110075 | 3542060250925.001 |
| | 6 | 2940609988400.000 | 2944368532074.996 | 3517555626025 | Infeasible |
| | 7 | 2915146338300 | 2925703262275 | Infeasible | Infeasible |
| (85000,195000) | 1 | 3318144585174.999 | 3318144585174.999 | 3318144585174.999 | 3318144585174.999 |
| | 2 | 3068756366824.997 | 3068756366825.000 | 319145948165.0005 | 3191459481650 |
| | 3 | 3012319474350 | 3012319474350 | 3167033548025.000 | 3173016027075 |
| | 4 | 2975911774624.995 | 2949036709200.005 | 3147030982999.999 | 3182687786700 |
| | 5 | 2945584579574.999 | 2949036709200.000 | 3144164110074.999 | 3542060250924.999 |
| | 6 | 2922659742824.999 | 2932222924250 | 3517555626025 | Infeasible |
| | 7 | 2897535391799.999 | 2925703262274.999 | Infeasible | Infeasible |

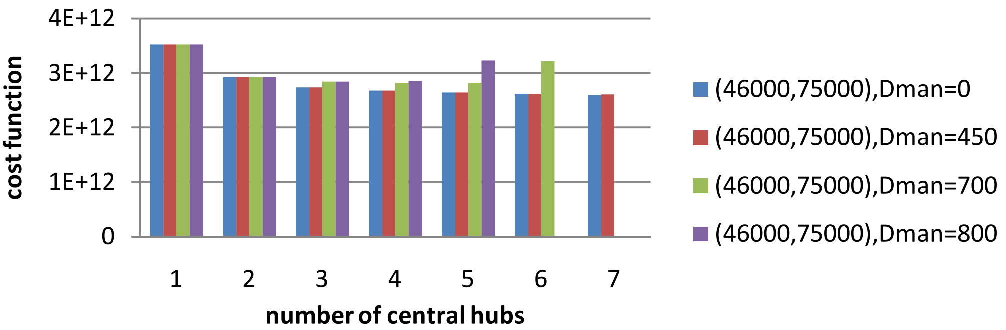


**Fig .2** The total cost function value for $(\alpha_H,\alpha_C)$=(0.7,0.6) in terms of the various value of $D_{man}$

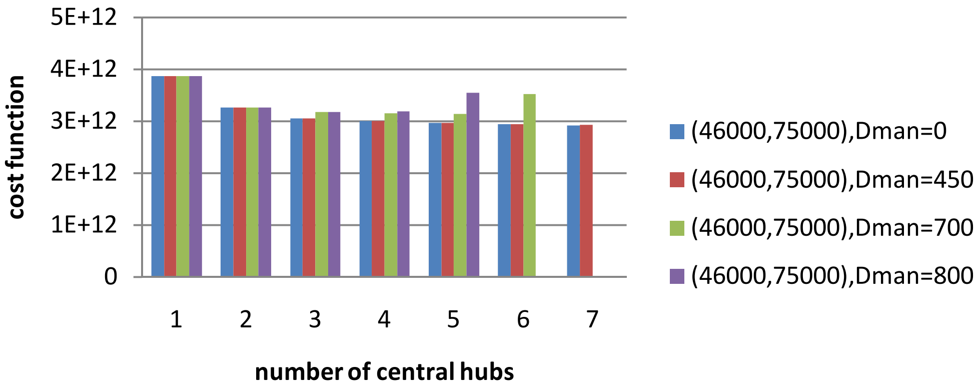


**Fig. 3** The total cost function value for $(\alpha_H,\alpha_C)=(0.8,0.7)$ in terms of the various value of $D_{man}$

It is obvious; cost function is lower, where the number of central hub is increased from star network to two-level network. At this point, the effect of mandatory dispersion of central hubs on the solution is studied, looking at the objective function value in tables for $(\alpha_H,\alpha_C)=(0.7,0.6)$, $(\alpha_H,\alpha_C)=(0.8,0.7)$ and various of coverage radius. In general, increasing $D_{man}$ value from 450 to 800, cost function value is increased due to enforcing mandatory dispersion to hierarchical network that results scatter between hubs and central hubs and leads to higher service level to demand nodes. Also figure .۲ and figure .۳ in terms of a fixed choice of $(r_H,r_C)=(46000,75000)$ and $p_C$ show that the rising trend in cost function is caused by the increase of $D_{man}$ but it is expected by increasing central hubs, cost function value is reduced even with the various value of $D_{man}$. The remarkable point is that, when $D_{man}=450$ in the increase of cost function value is not significance because the different between $D_{man}=0$ and $D_{man}=450$ is small and trivial for the hierarchical network. When $D_{man}=800$ and $p_C=6,7$ the problem resulted no feasible solution exists because the network proceeds to two-level network and this value of $D_{man}$ for central hub is not feasible as expected.

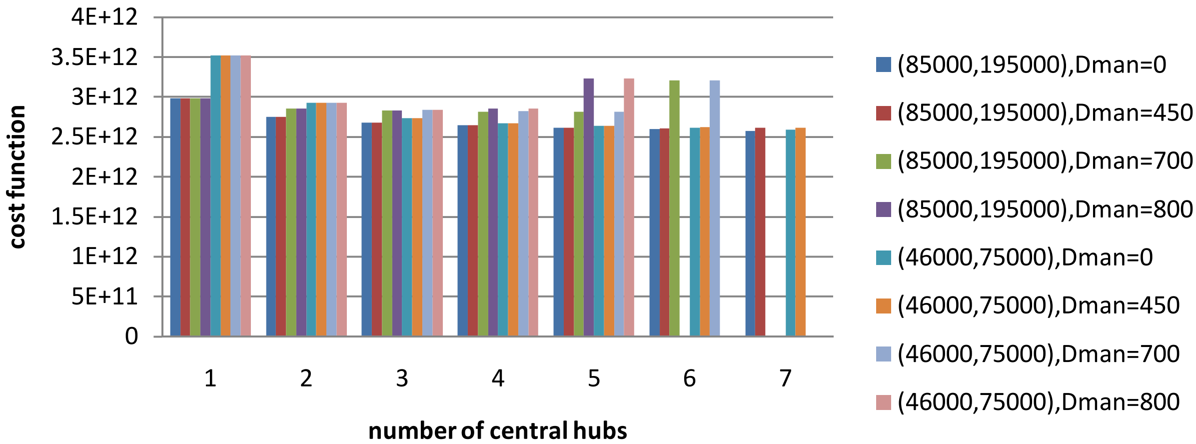


**Fig. 4** The total cost function value for the different of $D_{man}$ and covering radius and $(\alpha_H,\alpha_C)=(0.7,0.6)$

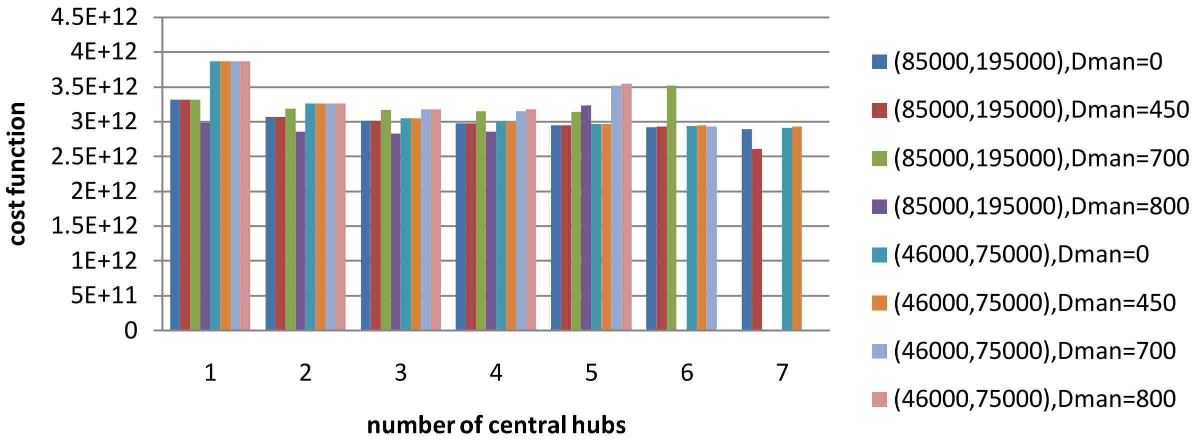


**Fig. 5** The total cost function value for the different of $D_{man}$ and covering radius and $(\alpha_H,\alpha_C)$=(0.8,0.6)

Another analysis is made about the effect of the various values of problem parameters. We observe that in all cases in table1 and table 2 for $(\alpha_H,\alpha_C)$=(0.7,0.6) and (0.8,0.7), results clearly show that for a fixed value of $P_C$, the various values of $D_{man}$, cost function value is increased by decreasing coverage radius of hubs and central hub from $(r_H,r_C)$=(85000,195000) to $(r_H,r_C)$=(46000,75000) is apparent the coverage radius constraints are enforced to network, hubs and central hubs are limited to serve special demand nodes ,which are in the nearest locations to hubs and central hubs, so cost function value is rising as figure .4 and figure .5 depict obviously our experiment for different values of $D_{man}$, coverage radius and $P_C$. Our test on paper model, the fact that for $P_C$=1, cost function value for a fixed choice of $(r_H,r_C)$, $(\alpha_H,\alpha_C)$ and the various of $D_{man}$ should be fixed. So that is one central hub and for $D_{man}$=0, 450, 700, 800 no changes is observed on cost function value.

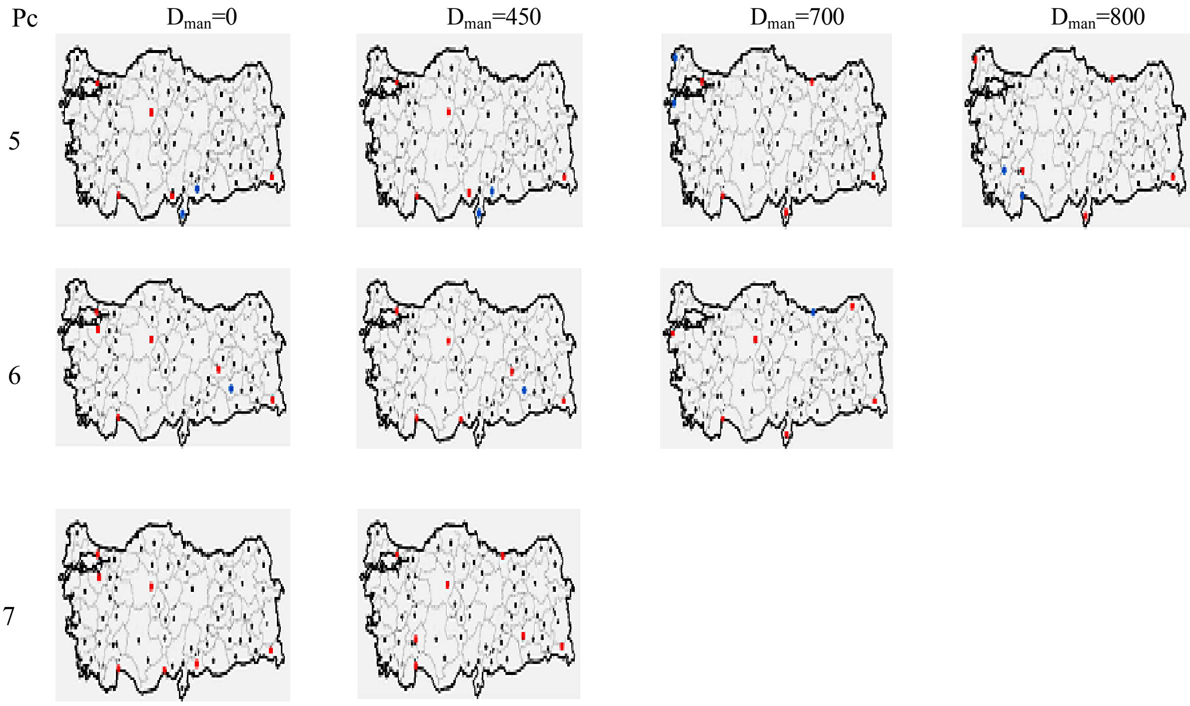


**Fig.6** comparing the location of hubs and central hubs with the different of $D_{man}$

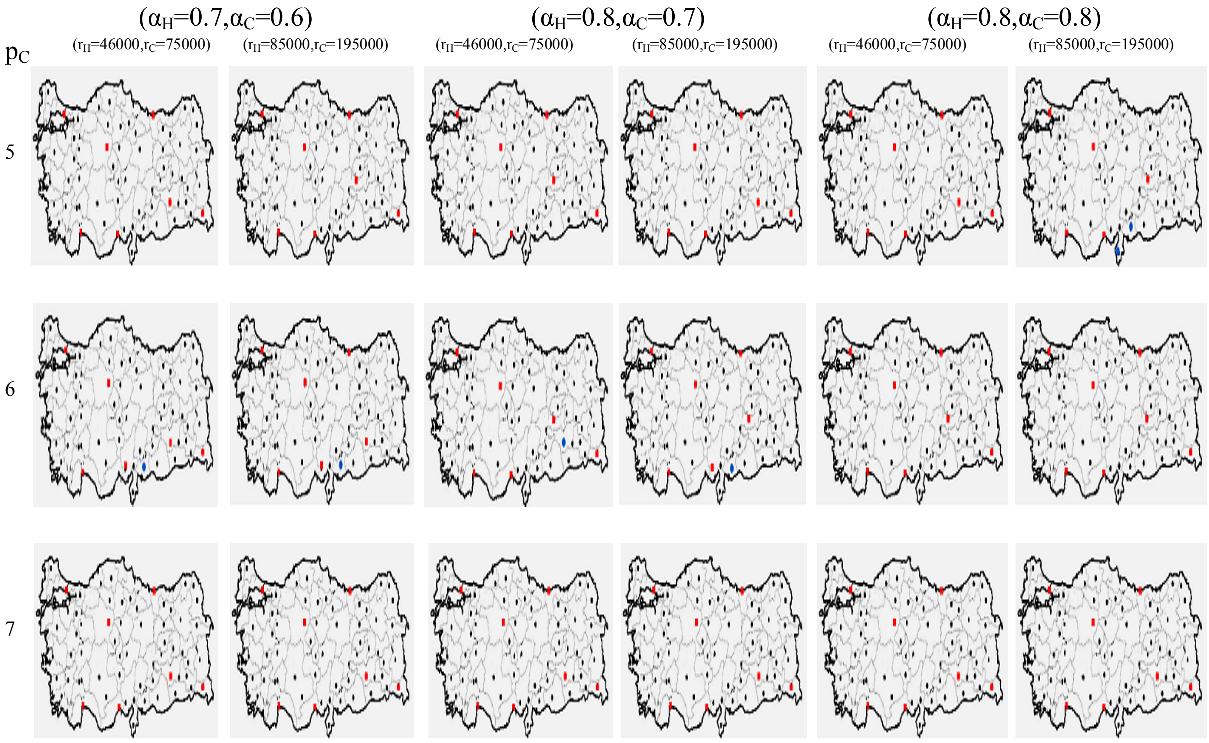


**Fig. 7** comparing the location of hubs and central hubs with the different of covering radius and $(\alpha_H,\alpha_C)$ and $D_{man}$=450

It is to be noted that, achieved findings for locating hubs and central hubs in Turkish dataset are given. We would like to investigate the effect of the number of $P_C$ and the various of $D_{man}$ and a fixed value of $(r_H,r_C)$=(46000,75000), $(\alpha_H,\alpha_C)$=(0.8,0.7), $P_H$=7, n=35 on the location of hubs and central hubs. The location of hubs and central hubs for Turkish dataset is reported on turkey map as shown in figure .6 and figure 7. Red square nodes are central hubs and blue circle nodes are hubs.

In this experiment, as shown in figure .6, when $D_{man}$=0 and $D_{man}$=450 are observed, the location of hubs and central hubs are not changed, where Adana, Antalya, Hakkari in the south of turkey, Ankara in center of turkey and Istanbul in the north west of turkey are central hubs, only G.Antep and Hatay remain hubs. For $D_{man}$=0 and $D_{man}$=450 central hubs and hubs remain the same, as expected, results obtained in table2 for $P_C$=5, $D_{man}$=0, 450 as the value of objective function are the same, so when $D_{man}$ is not imposed on the model, comparing with $D_{man}$ is applied on the model with $D_{man}$=450 for $P_C$=5, significance changes have not been observed on the location of central hubs and hubs. To see that, when a larger $D_{man}$ is imposed on the model for $P_C$=5 and Dman=700, the location of central hub is moved from the south of turkey to north-west of turkey and a mandatory distance of central hubs appears between them, explicity, one of the central hubs at Adana moves to Hattay and one located at Anakara moves to the north of turkey, Giresum and the rest of central hubs comparing with $D_{man}$=0, 450, $P_C$=5 remain the same. When $D_{man}$ is increased to 800, one located hubs was set at Antalya is replaced by Isparta, Antalya and Denizli become hubs and one of the other central hubs at Isparta moves to Edrine, it is obvious that when mandatory dispersion of central hubs has upward trend, we can see the most mandatory distance between central hubs and the location of central hubs scattered on one focused part of map, so service levels to another demand nodes are increased.

Further analysis reports that, in the increase of $P_C$, from 5 to 6, when there is no mandatory dispersion of central hubs restriction, most of the location of central hubs focus on center of turkey just one hub is located at Diyarbakir near Elazig as central hubs. Comparing to $D_{man}$=0 and $D_{man}$=450, the noticeable different on the location of central hubs is observed in two-level network. For $D_{man}$=700, $P_C$=6 and $D_{man}$=800, $P_C$=6, 7, the results are not feasible, therefore are not reported on map.

More analysis for locating of central hubs in this problem begins with different values of coverage radius and discount factors $\alpha_H$ and $\alpha_C$. As figure .7 depicts, when the dispersion restriction is satisfied on the model with a fixed choice of $D_{man}$=450, the effect of the various values of $(r_H,r_C)$=(46000,75000) and $(r_H,r_C)$=(85000,195000), significance changes on the location of central hubs are less than the effect of the different $D_{man}$ on the location of central hubs. When $P_C$=5, $(r_H,r_C)$ and $(\alpha_H,\alpha_C)$ are varied to the different of them, results suggest that only one location of central hub changes on turkey map. Results for $(\alpha_H,\alpha_C)$ =(0.8,0.7) are exactly alike with $(\alpha_H,\alpha_C)$=(0.8,0.8). As $P_C$=7 and the network is two-level network, results report that the location of central hubs for all $\alpha_H$=0.7, 0.8, $\alpha_C$=0.6, 0.7, 0.8 and $(r_H,r_C)$=(46000,75000) and $(r_H,r_C)$=(85000,195000) are the same cities, in this case, no changes are enforced on the location of central hubs.

An interesting analysis is carried out that to study how the problem parameters influence CPU time. Table3 and table4 report some scenario, including the various value of $\alpha_H$, $\alpha_C$, $P_C$ and $D_{man}$. When $\alpha_H$ and $\alpha_C$ are increased lead to in the increased of CPU time and $P_C$ is increased from 5 to 7 in the different values of $\alpha_H$, $\alpha_C$ and $D_{man}$, computational time is reduced. Adding mandatory dispersion of central hub constraint on the model in terms of $D_{man}$, $D_{man}$=800 take less time to solve the model, however infeasible solutions exist more for $D_{man}$=800. We conclude that when $D_{man}$ is increased from 0 to 800 considering the model parameters like as $\alpha_H$, $\alpha_C$, $P_C$ vary in any scenario, CPU time is reduced and the most different instances is for $D_{man}$=0 (mandatory dispersion in not imposed on the model) and the longest CPU time is reported for $D_{man}$=0 as mentioned, these results are explained for both $(r_H,r_C)$=(46000,75000) , $(r_H,r_C)$=(85000,195000), although the values of CPU time are different.

**Table 3** The computational times of the problem for $(r_H,r_C)$=(46000,75000)

| | | | CPU time(seconds) | | | |
|---|---|---|---|---|---|---|
| $\alpha_H$ | $\alpha_C$ | $P_C$ | $D_{man}$=0 | $D_{man}$=450 | $D_{man}$=700 | $D_{man}$=800 |
| 0.8 | 0.6 | 5 | 20.035 | 15.098 | 15.528 | 12.836 |
| 0.8 | 0.6 | 6 | 18.16 | 15.803 | 13.047 | Infeasible |
| 0.8 | 0.6 | 7 | 15.552 | 14.448 | Infeasible | Infeasible |
| 0.8 | 0.7 | 5 | 56.521 | 24.288 | 19.869 | 16.934 |
| 0.8 | 0.7 | 6 | 90.173 | 42.738 | 19.042 | Infeasible |
| 0.8 | 0.7 | 7 | 70.078 | 19.777 | Infeasible | Infeasible |
| 0.8 | 0.8 | 5 | 83.735 | 44.959 | 17.686 | 22.205 |
| 0.8 | 0.8 | 6 | 92.13 | 22.908 | 20.95 | Infeasible |
| 0.8 | 0.8 | 7 | 56.409 | 14.839 | Infeasible | Infeasible |
| 0.9 | 0.6 | 5 | 27.802 | 19.683 | 15.308 | 15.46 |
| 0.9 | 0.6 | 6 | 19.762 | 18.648 | 11.591 | Infeasible |
| 0.9 | 0.6 | 7 | 15.555 | 15.051 | Infeasible | Infeasible |
| 0.9 | 0.7 | 5 | 74.983 | 47.953 | 20.719 | 24.523 |
| 0.9 | 0.7 | 6 | 59.85 | 50.653 | 16.228 | Infeasible |
| 0.9 | 0.7 | 7 | 69.406 | 20.139 | Infeasible | Infeasible |
| 0.9 | 0.8 | 5 | 160.397 | 46.868 | 18.459 | 21.479 |
| 0.9 | 0.8 | 6 | 68.415 | 45.237 | 16.192 | Infeasible |
| 0.9 | 0.8 | 7 | 56.807 | 14.721 | Infeasible | Infeasible |

**Table 4** The computational times of the problem for $(r_H,r_C)$=(85000,195000)

| | | | CPU time(seconds) | | | |
|---|---|---|---|---|---|---|
| $\alpha_H$ | $\alpha_C$ | $P_C$ | $D_{man}$=0 | $D_{man}$=450 | $D_{man}$=700 | $D_{man}$=800 |
| 0.8 | 0.6 | 5 | 27.884 | 16.803 | 17.448 | 14.191 |
| 0.8 | 0.6 | 6 | 30.007 | 16.375 | 13.839 | Infeasible |
| 0.8 | 0.6 | 7 | 22.36 | 15.259 | Infeasible | Infeasible |
| 0.8 | 0.7 | 5 | 145.575 | 68.399 | 49.387 | 56.063 |
| 0.8 | 0.7 | 6 | 184.674 | 51.606 | 25.579 | Infeasible |
| 0.8 | 0.7 | 7 | 80.669 | 41.952 | Infeasible | Infeasible |
| 0.8 | 0.8 | 5 | 202.737 | 94.249 | 70.451 | 52.067 |
| 0.8 | 0.8 | 6 | 183.924 | 53.921 | 9.691 | Infeasible |
| 0.8 | 0.8 | 7 | 118.906 | 24.418 | Infeasible | Infeasible |
| 0.9 | 0.6 | 5 | 47.765 | 19.165 | 21.249 | 14.837 |
| 0.9 | 0.6 | 6 | 24.721 | 20.639 | 14.249 | Infeasible |
| 0.9 | 0.6 | 7 | 21.598 | 14.75 | Infeasible | Infeasible |
| 0.9 | 0.7 | 5 | 173.038 | 73.752 | 56.574 | 57.938 |
| 0.9 | 0.7 | 6 | 153.035 | 55.23 | 25.351 | Infeasible |
| 0.9 | 0.7 | 7 | 79.26 | 35.814 | Infeasible | Infeasible |
| 0.9 | 0.8 | 5 | 297.356 | 78.977 | 55.599 | 58.01 |
| 0.9 | 0.8 | 6 | 195.658 | 55.825 | 52.287 | Infeasible |
| 0.9 | 0.8 | 7 | 119.93 | 21.146 | Infeasible | Infeasible |

## 4 Conclusion

In our study, we have studied the hierarchical hub network enforcing hub and central hub covering radius and mandatory dispersion among central hubs that provide high service level for organization in a competitive environment.

A computational study using the Turkish dataset showed the proposed model sensitivity to its parameters like as $D_{man}$, coverage radius, the number of central hubs ($P_C$), discount factors as imposing $D_{man}$ leads to in the increase of cost function value whereas in the increase of coverage radius reduced cost function value, the effect of $D_{man}$, $(r_H,r_C)$ and $(\alpha_H,\alpha_C)$ in the various values of $P_C$, central hub moved to another location in order to create mandatory distance among central hubs for high services. Another outcome, adding the various values of $D_{man}$ in the different values of $(r_H,r_C)$ reduced computational times.

Considering more than one objective function in the fuzzy or stochastic environment, adding the other influential constraints into the model, and solving them by evolutionary algorithms can be proposed as a future research.

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