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Efficient DMUs Improvement Based on Input Expenses Reduction Through Using Data Envelopment Analysis

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Abstract Nowadays the main purpose in the models designed by Data Envelopment Analysis (DEA) is to improve the obtained outputs. In this method, which is expressed by Khodabakhshi, with an output-oriented BCC model, the output increases when the input increases. In this article we will discuss the efficient Decision Making Units (DMUs) in the input-oriented BCC model to reduce the input expenses significantly and wing the reduction of the produced outputs. Models are recommended based on DEA using turn-over with a constant return to scale to make efficient DMUs rival their references and reduce the input expenses to the level they can excel their reference DMUs. The model is efficient for DMUs with discretionary and non-discretionary inputs. Finally, numerical examples are presented to demonstrate the approach.

Keywords: DEA, Efficiency, Improvement, Input Expenses Reduction.

1 Introduction

Using the data envelopment analysis to estimate the relative efficiency is a wide comprehensive method. DEA is a non-parametric method to evaluate the relative efficiency of the Decision Making Units by inputs and outputs. This method which was primarily expressed by Charnes, et al., under the CCR model [1], has been improved by Banker, et al., under the so-called BCC model [2], which is increasingly used to measure the making units efficiency. DEA divides the decision making units into two groups of efficient and inefficient. Efficient units can be ranked using some models like Andersen and Petersen [3]. Some methods have been presented to improve inefficient units [4]. One of these methods to improve inefficiency of the DMUs is the output improvement one, which is presented by Khodabakhshi [5]. In this method, which is explained through the output-oriented BCC model, an increase in input is followed by an increase in output and ends to the DMUs ranking after the result changes. On one side, it is possible that the increase in inputs cause high expenses for the manufacturing companies, supposing the companies can't afford the expenses, it leads to the procedure proceeding by the past methods. Because all the manufacturing and industrial companies are seeking a method that decreases the input

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expenses, while the output is increased or at least fixed. The less the input expenses are, production lane manager can produce more flexibility in the inputs. Sometimes manufactures, to reduce the expenses, look for methods to alter outputs. It means methods that can decrease the input by changing the output.

Therefore, we, in the article, express the contrary of the model which is based on the BCC input oriented model and discretionary and non-discretionary models.

The advantage of these models is that, they put efficient DMUs in rival with their reference DMUs, as using revenue to a constant return to scale, reduces the being evaluated DMU outputs based on some determined quantity of subsidiary variables and result in remarkable reduction of the inputs. As mentioned, the less the input expenses one, manufactures can increase the variety in the outputs.

On the other side, these reductions should be in a way to increase the output, fix it or if they have to decrease the production, the output doesn't decrease so much, that our methods presents the third situation which is based on a little reduction in the output.

Different parts of the article are named as following: Basic DEA models are presented in the second part. The input expenses improvement model in the third part and finally in part four numerical examples are the results of the article are discussed.

2 Basic DEA Models

Suppose that there are n homological decision making units that each DMU_j $(j=1,2,...,n)$, includes m input, x_{ij} $(i=1,2,...,m)$, to produce s output, y_{rj} $(r=1,2,...,s)$. The input oriented BCC model is as following:

$$
\begin{aligned}
Min \quad & \theta-\varepsilon\left(\sum_{i=1}^{m}s_i^- +\sum_{r=1}^{s}s_r^+\right) \\
s.t. \quad & \sum_{j=1}^{n}\lambda_j x_{ij}+s_i^- =\theta x_{io}, && i=1,...,m, \\
& \sum_{j=1}^{n}\lambda_j y_{rj}-s_r^+ = y_{ro}, && r=1,...,s, \\
& \sum_{j=1}^{n}\lambda_j =1 \\
& \lambda_j \geq 0, && j=1,...,n, \\
& s_r^+ \geq 0, && r=1,...,s, \\
& s_i^- \geq 0, && i=1,...,m, \\
& \theta \ Free.
\end{aligned} \tag{1}
$$

On the other side, in the models similar to model (1) all the inputs and outputs are flexible according to the boss or the user's appeal. These variables are called discretionary variables. Non-discretionary variables are those which their change isn't in hands of the boss or the user. For instance, in calculating the turnover of the air force prey bases, the airplane flight is based on good condition of the weather. The condition of the weather can be supposed as an input, because the number of the successful missions or not successful mission as outputs are influenced by the weather condition. Even if it is non-discretionary, taking such inputs in account has a great importance to reflect the amount of efficiency.

The method has been presented by Banker and Murty as the following model:

$$
\begin{aligned}
Min \quad & \theta - \varepsilon\left(\sum_{i\in D} s_i^- + \sum_{r=1}^{s} s_r^+\right) \\
s.t. \quad & \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = \theta x_{io}, && i \in D, \\
& \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = x_{io}, && i \in ND, \\
& \sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = y_{ro}, && r = 1,\dots,s, \\
& \lambda_j \geq 0, && j = 1,\dots,n, \\
& s_r^+ \geq 0, && r = 1,\dots,s, \\
& s_i^- \geq 0, && i \in D, i \in ND, \\
& \theta \ Free.
\end{aligned}
\tag{2}
$$

In the model above, D shows the discretionary input and ND shows the non-discretionary input .Note that θ in the non-discretionary input isn't mentioned because ,their contraction is not in control of the decision maker. So all x_{io} in which $i \in ND$ are in a fixed amount.

3 The Input Improvement Method

In his article, Khodabakhshi, could increase the evaluated DMU output, using the input increase. But in all the manufacturing and industrial companies, the manufactures demand for a decrease in input expenses, even if this reduction is a vision and outputs don't increase. It is because the increase of expenses is so much that the production industry can't afford it.

We will design a form which will make a great reduction of the inputs with a little reduction in the outputs. For some determinants units that have a special usage, the reduction world lead to a better usage. And this is suitable for the determinant unit which can reduce the input expenses with a little reduction in the outputs. Our offering model is as follows:

$$
\begin{aligned}
Min \quad & \theta-\varepsilon\left(\sum_{i=1}^{m} s_i^- +\sum_{r=1}^{s} s_{r1}^+ -\sum_{r=1}^{s} s_{r2}^-\right) \\
s.t. \quad & \sum_{j=1}^{n}\lambda_j x_{ij}+s_i^- =\theta x_{io}, && i=1,\dots,m, \\
& \sum_{j=1}^{n}\lambda_j y_{rj}-s_{r1}^+ +s_{r2}^- =y_{ro}, && r=1,\dots,s, \\
& \sum_{j=1}^{n}\lambda_j =1, \\
& \lambda_j \geq 0, && j=1,\dots,n, \qquad (3)\\
& s_{r1}^+ \geq 0, && r=1,\dots,s, \\
& s_{r2}^- \geq 0, && r=1,\dots,s, \\
& s_i^- \geq 0, && i=1,\dots,m, \\
& \theta \ Free.
\end{aligned}
$$

Theorem 1. Model 3 is possible permanently.
Proof. Is sufficient $\lambda_j(\forall j\neq 0)=0, \lambda_0=1, \theta=1$ and $s_{r_2}^-, s_{r_1}^+, s_i^- =0, \forall r, \forall i$ so the model is possible.
Example. Consider four determinant DMU with an input and output [5].

Table 1 Data of DMUs

DMU	Input	Output
A	1	0.5
B	2	2
C	3	2
D	5	1

In this special example Khodabakhshi in fact has had the output of DMU_A, if increasing the input twice. But the same change in expenses may end to the manufactures dissatisfaction. Since in most cases the manager is not able to pay the huge expenses, he eventually has to persuade the past methods. Now consider model (3) with the above example.

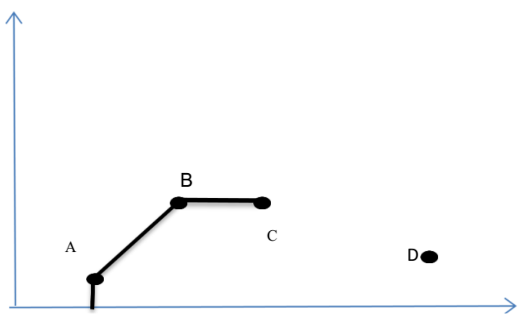

Fig. 1 Result from out of the main data Fig. 1

Table 2 the efficiency level in the input oriented BCC model

DMU	Efficiency	Reference-set
DMU_A	1	DMU_A
DMU_B	1	DMU_B
DMU_C	0.67	DMU_C
DMU_D	0.27	DMU_D

Table 3 the efficiency level using the model

DMU	Efficiency	Reference-set
DMU_A	1	DMU_A
DMU_B	0.5	DMU_A
DMU_C	0.34	DMU_A
DMU_D	0.20	DMU_A

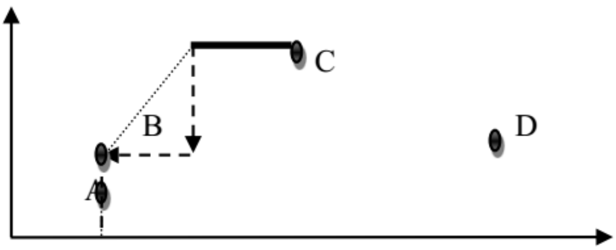

Fig 2 Result from out of changed data

So using the following changes the new input and output can be found:

$$Min \ \ \theta_B - \ \varepsilon(s^- + s_{11}^+ - s_{12}^- \ )$$

$$s.t. \qquad 2\lambda_A + 0.5\lambda_B + 2\lambda_C + \lambda_D + S^- - 0.5\theta_B = 0,$$

$$2\lambda_A + \lambda_B + 3\lambda_C + 5\lambda_D - S_{11}^+ + S_{12}^- = 1,$$

$$\lambda_A + \lambda_B + \lambda_C + \lambda_D = 1$$

$$S_{11}^-, S_{12}^+, S^+, \lambda_A.\lambda_B, \lambda_C, \lambda_D \geq 0.$$

So we have:

$$\theta_B^* = 0.5 \quad , S_{12}^- = 0.38 \quad , \lambda_A^* = 1$$

$$x_B = 2 \Rightarrow x_{B_{new}} = 2 \times 0.5 = 1$$

$$y_B = 2 \Rightarrow y_{B_{new}} = 2 - 0.38 = 1.62$$

As it is clear the input changes remarkably with a little reduction in output and DMU_B is placed on path its reference DMU_A. In fact, this model relatively meets the needs of the manager.

Now we can generalize the model in a way that the discretionary and non-discretionary inputs, which are as follows:

$$
\begin{aligned}
Min \quad & \theta - \varepsilon\left(\sum_{i\in D} s_i^- + \sum_{r=1}^{s} s_{r1}^+ - \sum_{r=1}^{s} s_{r2}^-\right) \\
s.t. \quad & \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = \theta x_{io}, && i \in D, \\
& \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = x_{io}, && i \in ND, \\
& \sum_{j=1}^{n} \lambda_j y_{rj} - s_{r1}^+ + s_{r2}^- = y_{ro}, && r = 1,\dots,s, \\
& \lambda_j \geq 0, && j = 1,\dots,n, \qquad (4)\\
& s_{r1}^+ \geq 0, && r = 1,\dots,s, \\
& s_{r2}^- \geq 0, && r = 1,\dots,s, \\
& s_i^- \geq 0, && i \in D, i \in ND, \\
& \theta \ Free.
\end{aligned}
$$

s_{r1}^+ and s_{r2}^- represent the maximum and minimum interval differentiations for the target function which is as follows:

If the s_{r1}^+ is positive, the rth output of the being evaluated unit will be increased as much as s_{r1}^+ amount.

If the s_{r2}^- is positive, the rth output of the being evaluated unit will be decreased as much as s_{r2}^- amount.

If $s_{r1}^+ = s_{r2}^- = 0$, rth output will remain without any changes.

The conditions of efficiency for DMU_o being evaluated are as below:

Definition1 . DMU_o under model (3) and (4) is efficient if:

1. $\theta_o^* = 1$
2. Optimal amount of all slacks are zero.

It is significant that the result model is solved in two stages: At first the improved level which is $\min\theta_o = \theta_o^*$ is calculated without considering the amount of all slacks. And then replacing θ_o with θ_o^*, the quantity of $MAX\left(\sum_{i\in D} s_i^+ + \sum_{r=1}^{s} s_{r1}^+ - \sum_{r=1}^{s} s_{r2}^-\right)$ is found.

Theorem 2. If DMU_o is efficient under models (3) and (4), then it is efficient under models (1) and (2) with input oriented. But its contrary is not necessarily correct.

Proof. Consider the previous example: DMU_A which is efficient under model (3) was also efficient in model (1). But DMU_B in the BCC oriented model is efficient whereas it is not efficient in model (3).

Theorem 3. Model (3) and (4) are bounded (the dual models are feasible) if and only if $\varepsilon_0 \le 1 \Big/ \sum_{i=1}^{m} x_{io}$

Proof. Suppose that to solve the model, the 2-stage method is used instead of the direct method. If so dual of the model to modify the ε interval would be as follows:

$$
\begin{aligned}
Max \quad & z = \sum_{r=1}^{s} u_r y_{ro} + v_o \\
s.t. \quad & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} \mu_i x_{ij} - v_o \le 0, && j = 1,\dots,n, \\
& \sum_{i=1}^{m} \mu_i x_{io} = 1, && (The\ dual\ input\ oriented\ BCC\ \ \text{mod}el) \\
& u_r = \varepsilon, && r = 1,\dots,s, \\
& \mu_i \ge \varepsilon, && i = 1,\dots,m.
\end{aligned} \tag{5}
$$

Consider $\varepsilon_0 = 0$; therefore, $v_o = \max_{l \le j \le n}(x_{lj}/x_{lo})$, $\mu_l = 1/x_{lo}$, $\mu_i = 0$, $\left(i = 1,2,\dots,m,\ i \ne 1\right)$, $u_r = 0$ $\left(r = 1,2,\dots,s\right)$ are feasible solution for dual model. Then the dual model is feasible. Now suppose that $\varepsilon = \bar{\varepsilon}$ and $(\bar{u}, \bar{\mu}, \bar{v}_o)$ are the arbitrary feasible solution for the dual problem. Therefore, if $\bar{\varepsilon} \le \bar{\mu}_i$ for $\left(i = 1,2,\dots,m\right)$,and $\bar{\varepsilon} \sum_{i=1}^{m} x_{io} \le \sum_{i=1}^{m} \bar{\mu}_i x_{io} = 1$, in other word the necessary condition is being bounded of the envelopment form. It means, if $\bar{\varepsilon} \le 1 \Big/ \sum_{i-1}^{m} x_{io}$, then $\bar{\mu}_l = 1 \Big/ \sum_{i=1}^{m} x_{io}$ for $\left(i = 1,2,\dots,m\right)$ and $\bar{u}_r = \bar{\varepsilon}$ for$\left(r = 1,2,\dots,s\right)$, we have a feasible solution for dual program by choosing v_o as a sufficient large number. So the envelopment problem will be bounded [6].

4 Numerical Examples

In this section we represent our method using a numerical example. In this example 12 decision making units (DMUs) have been shown with two inputs (x_1, x_2) and two outputs (y_1, y_2) in table 3.

We have calculated efficiency level of each DMU with two BCC methods in the input oriented and discretionary and non-discretionary models in table 4. Then in table 5 and 6, we have found efficiency levels using models (3) and (4).

Table 4 Shown Inputs and Outputs

DMUs	Input1	Input2	Output1	Output2
1	30	151	100	90
2	19	131	150	50
3	25	160	160	55
4	27	168	180	72
5	22	158	94	66

DMUs	Input1	Input2	Output1	Output2
6	55	255	230	90
7	33	235	220	88
8	31	206	152	80
9	30	244	190	100
10	50	268	250	100
11	53	306	160	147
12	38	248	250	120

Table 5 the efficiency level under models (1) and (2) and the reference set

DMU	Efficiency (BCC)	Reference set	Efficiency (D,ND)	Reference set
1	1	DMU_1	1	DMU_1
2	1	DMU_2	1	DMU_2
3	0.89	$DMU_2 - DMU_4$	0.84	$DMU_2 - DMU_7$
4	1	DMU_4	1	DMU_4
5	0.87	$DMU_1 - DMU_2$	1	DMU_5
6	0.93	$DMU_1 - DMU_4$	0.67	$DMU_7 - DMU_{12}$
7	1	$DMU_2 - DMU_4 - DMU_{12}$	1	DMU_7
8	0.8	$DMU_1 - DMU_4 - DMU_{11} - DMU_{12}$	0.75	$DMU_1 - DMU_2 - DMU_{12}$
9	0.98	$DMU_1 - DMU_2 - DMU_{12}$	0.99	$DMU_1 - DMU_2 - DMU_{11}$
10	1	DMU_{10}	1	DMU_{10}
11	1	DMU_{11}	1	DMU_{11}
12	1	DMU_{12}	1	DMU_{11}

Table 6 the efficiency level under our recommended model (3)

DMU	Efficiency(BCC)	Reference set
1	0.94	DMU_2
2	1	DMU_2
3	0.82	DMU_2
4	0.80	DMU_2
5	0.84	DMU_2
6	0.56	DMU_2
7	0.62	DMU_2
8	0.65	DMU_2
9	0.66	DMU_2
10	0.55	DMU_2
11	0.51	DMU_2
12	0.57	DMU_2

As you can see the numbers of the efficient DMUs are reduced. For instance consider DMU_1 : We want to know if the DMU would receive to its reference which is DMU_2 and with which changes it would?

$$\theta^* = 0.94 \ , s_{i1} = 0 \ , s_{i2} = 1.02 \ , s_{r1}^+ = 4.17 \ , s_{r2}^+ = 0 \ , s_{r1}^- = 0 \ , s_{r2}^- = 3.32.$$

$$x_1 = (20,151)\begin{cases} x_{1_{new}} = 20 \times 0.94 = 18.8 \\ x_{2_{new}} = 151 \times 0.94 = 140.96 \end{cases} \Rightarrow x_{1_{new}} = (18.8, 140.96)$$

$$y_1 = (100,90)\begin{cases} y_{1_{new}} = 100 + 4.17 = 104.17 \\ y_{2_{new}} = 90 - 3.33 = 86.67 \end{cases} \Rightarrow y_{1_{new}} = (104.17, 86.67)$$

Note that (x_1, y_1) increase or decrease, just according to the recommended model and based on the manager's tendency.

Table 7 the altered variable for model (4)

DMU	Efficiency (D,ND)	Reference set
1	0.96	DMU_2
2	1	DMU_2
3	0.75	DMU_2
4	0.68	DMU_2
5	0.88	DMU_2
6	0.29	DMU_2
7	0.52	DMU_2
8	0.60	DMU_2
9	0.59	DMU_2
10	0.31	DMU_2
11	0.25	DMU_2
12	0.42	DMU_2

When using model (4), the DMUs outputs are exactly equal to the reference DMUs Outputs and they exactly matches their reference section.

5 Conclusions

In 2007, the output improvement model was expressed by Khodabakhshi [5] in which an increase in an input was followed by an increase in an output. In most cases, the manager is not able to pay the huge expenses, and he/she eventually has to persuade the past methods. But because a lot of companies and manufacturers are seeking a model to decrease the input, in this article we have based our method on the basis of input improvement. In this article, we have discussed efficient DMUs in the input-oriented BCC models and discretionary and non-discretionary ones; with a little reduction in the output, the inputs have reduced significantly. One of the points of the model compared to Khodabakhshi's model is that in addition to the significant reduction of the inputs, which is demanded by lots of industrial manufacturers, non- discretionary inputs in which contraction is not manipulated by the manager can play an effective role in the improvement. Both procedures have been proofed by numerical examples. Eventually, studying and using the models in industrial researches will end to economize in using inputs as goods and producing balanced outputs.

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