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A new solving approach for fuzzy multi-objective programming problem in uncertainty conditions by using semi-infinite linear programing

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Abstract In practice, there are many problems which decision parameters are fuzzy numbers, and some kind of this problems are formulated as either possibilitic programming or multi-objective programming methods. In this paper, we consider a multi-objective programming problem with fuzzy data in constraints and introduce a new approach for solving these problems base on a combination of the multi-objective programming model and a linear semi-infinite programming problem. In particular, in the suggested solving process, we will use the weighted method to solve the mentioned multi-objective programming problem. Finally, a numerical example is included to illustrate the suggested solving process.

Keyword: Linear Programming, Multi-objective Linear Programming Problem, Fuzzy Programming, Semi-Infinite Linear Programming.

1 Introduction

The main idea of fuzzy sets was first proposed by Zadeh in 1970, as a mean of handling uncertainty that is due to rather than to randomness. After Zadeh's pioneering works, many valuable works are appeared in the literature. For example, Ramik and Rimanek [8] have also dealt with problem with fuzzy parameters in the constraints. Campos and Verdegay [1] studied linear programming problem with fuzzy constraints and coefficients in both matrix and right hand side of the constraints sets. Kumar [6] considered a multi-objective two person zero-sum matrix game with fuzzy goals, assuming that each player has a fuzzy goal for each of the payoffs. The max-min solution is formulated for this multi-objective game model. Ren et als. [9] in their study, administrative, economic and ecological benefits were regarded as the planning objectives. Moreover, the variations of irrigation water demand with rainfall, soil moisture content and evapotranspiration were considered in the developed model. Optimal irrigation plans were obtained under different possibility levels of fuzzy parameters. Nasseri and Zavieh presented a new method to solve a fuzzy linear programming problem with

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fuzzy coefficients in the constraints and the objective function based on solving an associated multi-objective model [7]. Simić et al. [10] proposed an interval-parameter semi-infinite programming model for used tire management and planning. The underlying difference between this model and those developed in previous research is its ability to consider the effects of external impact factors related to complicated economic, environmental, and social activities on used tire management systems. Wu et al. have used an analytic center based on cutting plane method to solve linear semi-infinite programming problems [11]. Goberna et al. [5] analyzed the effect on the optimal value of a given linear semi-infinite programming problem of the kind of perturbations which more frequently arise in practical applications. Fang et al. [4] proposed a linear programming with fuzzy coefficients in constraints. In this study, we consider a multi-objective problem with fuzzy coefficients in constraints and then present a new model for solving FLSIP problem. For this intent, Section 2 outlines the basic fuzzy concepts required for the next sections. Section 3 is concerned to introducing of the linear programming problem with fuzzy coefficients. In this subsequently, we define a Multi-Objective Linear Semi-Infinite Programming (MOLSIP) problem. Finally in Section 4, we present a numerical example for the proposed method.

2. Preliminaries

Here, we first give some fundamental concepts of fuzzy sets which are directly related to our discussion in this paper and taken from [2, 3, 7].

Definition 2.1. Let $\mathbb{R}$ be the real line. A fuzzy set $\tilde{A}$ in $\mathbb{R}$ is defined to be a set of ordered pairs $\tilde{A}=\left\{\left(x,\mu_{\tilde{A}}\left(x\right)\right):x\in\mathbb{R}\right\}$, where $\mu_{\tilde{A}}\left(x\right)$ is called the membership function for the fuzzy set. The membership function maps each element of $\mathbb{R}$ to a membership value between 0 and 1.

Definition 2.2. Assume $\tilde{A}$ is a fuzzy set and $\alpha\in\left(0,1\right]$, then an α-cut of $\tilde{A}$ is defined as $\left\{x\,|\,x\in\mathbb{R},\ \mu_{\tilde{A}}\left(x\right)\geq\alpha\right\}$, and we briefly denote it as $\tilde{A}_{\alpha}$.

We denote the set of all triangular fuzzy numbers on $\mathbb{R}$ by $F\left(\mathbb{R}\right)$.

Since fuzzy ordering has the key role for solving fuzzy mathematical models, hence in this study we follow the suggested approach by Fang et al. in [5] (see also in [7]).

Definition 2.3. If $\tilde{A}_1,\tilde{A}_2\in F\left(\mathbb{R}\right)$, and $\alpha\in\left(0,1\right]$, then $\tilde{A}_1\geq_{\alpha}\tilde{A}_2$ if and only if

$$L_{A_1}\left(t\right)\geq_{\alpha}L_{A_2}\left(t\right),$$
$$R_{A_1}\left(t\right)\geq_{\alpha}R_{A_2}\left(t\right),\qquad\forall t\in\left(\alpha,1\right],$$

where L is the lower bound of α-cut and R is the upper bound of α-cut for fuzzy numbers.

3. Proposing method

3.1 Multi-objective problem with fuzzy data

Now we consider a multi-objective programming problem with fuzzy data in constraint based on the ordering definition which is given in Definition 2.3:

$$Min\left\{f_1,\ldots,f_k\right\}$$

$$\text{s.t:} \quad \sum_{j=1}^{n} a_{ij} x_j \geq_{\alpha} b_i, \qquad i = 1, \ldots, q, \tag{1}$$

$$x_j \geq 0, \qquad j = 1, \ldots, n,$$

where $c_j, a_{ij}, b_i \in F(\mathbb{R}), i = 1, \ldots, q,\ j = 1, \ldots, n.$

According to Definition 2.3, Problem (1) can be changed to Problem (2):

$$Min\{f_1, \ldots, f_k\}$$

$$\text{s.t:} \quad \sum_{j=1}^{n} L_{a_{ij}}(t) x_j \geq_{\alpha} L_{b_i}(t) \tag{2}$$

$$\sum_{j=1}^{n} R_{a_{ij}}(t) x_j \geq_{\alpha} R_{b_i}(t) \qquad \forall t \in [\alpha \quad]$$

$$x_j \geq 0, \quad j = 1, \ldots, n.$$

3.2 A cutting-plan solution method

In this section, we propose a new approach for solving Multi-Objective Linear Programming (MOLP) problems with fuzzy data in constraints. In the process of solving MOLP, we need to solve a Linear Semi-Infinite Programing (LSIP) problem. We denote the achieved problem as MOLSIP in the abbreviated form.

Problems (1) and (3) are equivalent:

$$Min\{f_1, \ldots, f_k\} \tag{3}$$

$$\text{s.t:} \quad \begin{pmatrix} f_{11}(t_1) & \cdots & f_{1n}(t_1) \\ \vdots & \ddots & \vdots \\ f_{w1}(t_w) & \cdots & f_{wn}(t_w) \end{pmatrix} \begin{pmatrix} x_1 \\ . \\ . \\ . \\ x_n \end{pmatrix} \leq \begin{pmatrix} b_1(t_1) \\ . \\ . \\ . \\ b_w(t_w) \end{pmatrix}, \quad \forall t_i \in T, i = 1, \ldots, w,$$

$$x_j \geq 0, \quad j = 1, \ldots, n,$$

where **T** is a compact metric space, $f_{pj}(t), b_p(t),\ p = 1, \ldots, w,\ j = 1, \ldots, n$ are real-valued continuous functions on **T.** We consider

- $f_{ij}(t) = L_{a_{ij}}(t) \quad i = \quad \ldots\ q,\ \ f_{ij}(t) = R_{a_{i-q\,j}}(t) \quad i = q + \quad \ldots \quad q$ and $j = 1, \ldots, n,$
- $b_i(t) = L_{b_i(t)},\ i = 1, \ldots, q, b_i(t) = R_{b_{i-q}}(t) \quad i = q + \quad \ldots \quad q$ and $j = 1, \ldots, n,$
- $w = 2q,\ t \in [\alpha, 1],\ \alpha \in (0, 1].$

Problem (3) is a Multi-Objective Linear Semi-Infinite Programming (MOLSIP) problem with n variables and infinitely many constraints. We will use here the "cutting plane approach" for solving linear semi-infinite programming problems. Now by using the concepts of cutting plan approach, we can easily design an iterative algorithm that adds w constraints at a time until an optimal solution is identified. To be more specific, at the k^{th} iteration, given

$T_k = \{t^1, t^2, \ldots, t^k\}$, where $t^k = (t_1^k, \ldots, t_w^k) \in T^w$, and $k \geq 1$. We consider the following Multi-Objective Linear Programming ($MOLP$) problem as:

$$Min\{f_1, \ldots, f_k\} \tag{4}$$

$$\text{s.t:} \begin{pmatrix} f_{11}(t_1^1) & \cdots & f_{1n}(t_1^1) \\ \vdots & \ddots & \vdots \\ f_{w1}(t_w^1) & \cdots & f_{wn}(t_w^1) \\ - & - & - \\ & \vdots & \\ & . & \\ - & - & - \\ f_{11}(t_1^k) & \cdots & f_{1n}(t_1^k) \\ \vdots & \ddots & \vdots \\ f_{w1}(t_w^k) & \cdots & f_{wn}(t_w^k) \end{pmatrix} \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} \geq \begin{pmatrix} b_1(t_1^1) \\ \vdots \\ b_w(t_w^1) \\ - \\ \vdots \\ - \\ b_1(t_1^k) \\ \vdots \\ b_w(t_w^k) \end{pmatrix},$$

$$x_j \geq 0, \quad j = 1, \ldots, n.$$

Let R^k be the feasible region of $MOLP$ problem which is defined in (4). If $x^k = (x_1^k, x_2^k, \ldots, x_n^k)$ is a pareto optimal solution for Problem (4), then consider the "constraint violation functions" as follows:

$$v_p^{k+1}(t) = \sum_{j=1}^{n} f_{pj}(t) x_j^k - b_p(t), \quad \forall t \in T, \; p = 1, \ldots, w .$$

Since $f_{pj}(t)$ and $b_p(t)$ are continuous over **T**, and also **T** is a compact set, then the function $v_p^{k+1}(t)$ achieves its minimum over **T**, for $p = 1, \ldots, w.$ Let t_p^{k+1} be such a minimizer and consider the value of $v_p^{k+1}(t_p^{k+1})$, for $p = 1, \ldots, w$. If the value is greater than or equal to zero, for $p = 1, \ldots, w$, then $x^k = (x_1^k, x_2^k, \ldots, x_n^k)$ becomes a feasible solution of $MOLP$, and hence, $x^k = (x_1^k, x_2^k, \ldots, x_n^k)$ is optimal for LSIP, because the feasible region R^k of Problem (4) is not smaller than the feasible region of LSIP.

The following theorem will be useful in our numerical discussion as well as given in [5] and we omit the proof here.

Theorem 3.1. Let $\{x^k\}$ be a sequence which is generated by cutting plan algorithm, if there exist an $M > 0$ such that $x^k \leq M, \forall k,$ then there is a subsequence which converges to an optimal solution of LSIP.

1. Numerical Example

For illustration of our suggested approach, here we consider a multi-objective problem with triangular fuzzy number for coefficients in constraint and each of object function have same price ($w_1 = w_2 = 0.5$).

$$Max\, Z_1 = -x_1 - 3x_2$$
$$Max\, Z_2 = 1.5x_1 + 2.5x_2$$

$$\begin{aligned} s.t: \quad & (-1.2,-1,-0.7)x_1+(1.5,2,2.4)x_2 \le_\alpha (16,18,19),\\ & (3.7,4,4.5)x_1+(2.6,3,3.3)x_2 \le_\alpha (39,40,42),\\ & (2.5,3,3.2)x_1+(0.8,1,1.6)x_2 \le_\alpha (23,25,27),\\ & x_1,x_2 \ge 0\,. \end{aligned}$$

In this example, we assume that $\alpha=0.6$ and also $t^1=\left(t_1^1,t_2^1,t_3^1,t_4^1,t_5^1,t_6^1\right)=\left(0.6,0.7,0.8,0.6,0.7,0.8\right)$ as an arbitrary point. Then we have

$$Max\, Z_1=-x_1-3x_2$$
$$Max\, Z_2=1.5x_1+2.5x_2$$
$$\text{S.t:}\ \begin{pmatrix} 0.2t_1^1-1.2 & 0.5t_1^1+1.5\\ 0.3t_2^1+3.7 & 0.4t_2^1+2.6\\ 0.5t_3^1+2.5 & 0.2t_3^1+0.8\\ -0.3t_4^1-0.7 & -0.4t_4^1+2.4\\ -0.5t_5^1+4.5 & -0.3t_5^1+3.3\\ -0.2t_6^1+3.2 & -0.6t_6^1+1.6 \end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix} \le \begin{pmatrix} 2t_1^1+16\\ t_2^1+39\\ 2t_3^1+23\\ -t_4^1+19\\ -2t_5^1+42\\ -2t_6^1+27 \end{pmatrix},\quad \forall t_i\in[\alpha,1],$$
$$x_1,x_2\ge 0.$$

By solving the current multi-objective linear programming problem, for every objective function, we have:

$$Z_1^*=0\, and\; x^1=\left(x_1^1,x_2^1\right)=\left(0,0\right)$$
$$Z_2^*=26.515\, and\; x^1=\left(x_1^1,x_2^1\right)=\left(3.409,8.561\right)$$

- For problem Z_1 with $x^1=\left(x_1^1,x_2^1\right)=\left(0,0\right)$:

  Since $v_1^2\left(t_1\right)=17.2$, $v_2^2\left(t_2\right)=39.6$, $v_3^2\left(t_3\right)=24.2$, $v_4^2\left(t_4\right)=18$, $v_5^2\left(t_5\right)=40$ and $v_6^2\left(t_6\right)=25$ are ≥ 0. The minimizers of $v_1^2\left(t_1\right)$, $v_2^2\left(t_2\right)$, $v_3^2\left(t_3\right)$, $v_4^2\left(t_4\right)$,$v_5^2\left(t_5\right)$ and $v_6^2\left(t_6\right)$ over $\left[\alpha,1\right]$ are (0.6,0.6,0.6,1,1,1), respectively. The algorithm stops and the optimal solution is $x^*=x^1=\left(0,0\right)$.

- For problem Z_2 with $x^1=\left(x_1^1,x_2^1\right)=\left(3.409,8.561\right)$:

  Since $v_1^2\left(t_1\right)=4.29$, $v_2^2\left(t_2\right)=0.69$, $v_3^2\left(t_3\right)=6.22$, $v_4^2\left(t_4\right)=2.9$, $v_5^2\left(t_5\right)=3.46$ and $v_6^2\left(t_6\right)=3.87$ are nonnegative and the minimizers of $v_1^2\left(t_1\right)$, $v_2^2\left(t_2\right)$, $v_3^2\left(t_3\right)$, $v_4^2\left(t_4\right)v_5^2\left(t_5\right)$ and $v_6^2\left(t_6\right)$ over $\left[\alpha,1\right]$ are (1,1,1,0.6,0.6,0.6), respectively. The algorithm stops and the optimal solution is $x^*=x^1=\left(3.409,8.561\right)$.

So, we have the following problem:

$$Max\;\; w_1\left(-x_1-3x_2\right)+w_2\left(1.5x_1+2.5x_2\right)=\frac{1}{2}\left(1.5x_1+2.5x_2\right)$$
$$\begin{aligned} s.t: \quad & -1.08x_1+1.8x_2\le 17.2,\\ & 3.91x_1+2.88x_2\le 39.7,\\ & 2.9x_1+0.96x_2\le 24.6,\\ & -0.88x_1+2.64x_2\le 19.6, \end{aligned}$$

$$4015x_1 + 3.09x_2 \leq 40.6,$$
$$3.04x_1 + 1.12x_2 \leq 25.8,$$
$$x_1, x_2 \geq 0 .$$

So, the optimal solution of the main problem is $Z^* = 13.25$ with $x^* = (3.409, 8.561)$.

5. Conclusion

In this paper, we investigated a linear programming problem with fuzzy data, and as a solving process, we proposed a new model denoted as "Weighted Method" for the concluded MOLSIP problem. By using the α -preference, we used the cutting plane algorithm to solve the reduced semi-infinite linear programming problem. Finally, we illustrate our model by numerical example.

Reference

1. L. Campos and J.L. Verdegay, Linear programming problems and ranking of fuzzy numbers, Fuzzy Sets and Systems, 32 (1989) 1-11.
2. B.Y. Cao, Optimal Models and Methods with Fuzzy Quantities, Studies in Fuzziness and Soft Computing, volume 248, Springer-verlag, Berlin, Heidelberg,(2010).
3. D. Dubois and H. Prade, Fuzzy Sets and Systems: Theory and Applications, Academic Press, New York, (1980).
4. S.C. Fang, C.F. Hu, H.F. Wang and S.Y. Wu, Linear programming with fuzzy coefficients in constraints, Computers and Mathematics with Applications, 37 (1999) 63-76.
5. M.A. Goberna, S. Gómez, F. Guerra and M.I. Todorov, Sensitivity analysis in linear semi-infinite programming: perturbing cost and right-hand-side coefficients. European Journal of Operational Research, 181 (2007) 1069-1085.
6. S. Kumar, max-min solution approach for multi -objective matrix game with fuzzy goals, Yugoslav Journal of Operations Research, 26 (2016) 51-60.
7. S.H. Nasseri, and H. Zavieh, A multi-objective method for solving fuzzy linear programming based on semi-infinite models, Fuzzy Information and Engineering, 10 (2018), 95-102.
8. J. Ramilk and J. Rimanek, Inequality relation between fuzzy numbers and its use in fuzzy optimization, Fuzzy Sets and Systems, 16 (1985) 123–138.
9. C. Ren, P. Guo, Q. Tan and L. Zhang, A multi-objective fuzzy programming model for optimal use of irrigation water and land resources under uncertainty in Gansu Province, China, Journal of Cleaner Production, 164 (2017) 85-94.
10. V. Simić, S. Dabić-Ostojić and N. Bojović, Interval-parameter semi-infinite programming model for used tire management and planning under uncertainty, Computers & Industrial Engineering, 113 (2017) 487-501.
11. S.Y. Wu, S.Ch. Fang and Ch.J. Lin, Analytic center based cutting plane method for linear semi-infinite programming. In López, M.A. Semi-Infinite Programming: Recent Advances. Kluwer, Dordrecht,(2001) 221–233.