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Some properties of Pythagorean fuzzy ideal of near-rings

A.K. Adak[1], D. Darvishi Salokolaei[*,2]

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Abstract Theory of Pythagorean fuzzy sets possesses significant advantages in handling vagueness and complex uncertainty. Additionally, Pythagorean fuzzy information is useful to simulate the ambiguous nature of subjective judgments and measure the fuzziness and imprecision more flexibly. The aim of this research is to develop the concept of the Pythagorean fuzzy ideal of a near ring and discusses their desirable properties. Based on the ideas, this paper introduces a novel concept of homomorphism of pythagorean fuzzy ideals of near rings.

Keywords: Intuitionistic Fuzzy Set, Pythagorean Fuzzy Set, Pythagorean Fuzzy Ideals, Cut Set, Homomorphism Near-rings.

1 Introduction

Zadeh [1] introduced the idea of fuzzy set which has a membership function, μ that assigns to each element of the universe of discourse, a number from the unit interval [0, 1] to indicate the degree of belongingness to the set under consideration. The notion of fuzzy sets generalizes the theory of classical sets by allowing intermediate situations between the whole and nothing. In a fuzzy set, a membership function is defined to describe the degree of membership of an element to a class. The membership value ranges from 0 to 1, where 0 shows that the element does not belong to a class, 1 means belongs, and other values indicate the degree of membership to a class. For fuzzy sets, the membership function replaced the characteristic function in crisp sets. Since the pioneering work of Zadeh, the fuzzy set theory has been used in different disciplines such as management sciences, engineering, mathematics, social sciences, statistics, signal processing, artificial intelligence, automata theory, medical and life sciences.

The concept of fuzzy sets theory seems to be inconclusive because of the exclusion of nonmembership function and the disregard for the possibility of hesitation margin. Atanassov [2] critically studied these short comings and proposed a concept called intuitionistic fuzzy sets (IFSs). The construct (that is, IFSs) incorporates both membership function, μ and nonmembership function, ν with hesitation margin, π (that is, neither membership nor

[*] **Corresponding Author.** (✉)
E-mail: d_darvishi@pnu.ac.ir (D. Darvishi)

A.K. Adak
Assistant Professor, Department of Mathematics, Ganesh Dutt College, Begusarai, India. (amaladak17@gmail.com)

D. Darvishi
Assistant Professor, Department of Mathematics, Payame Noor University, Tehran, Iran.

nonmembership functions), such that $\mu+\nu\leq 1$ and $\mu+\nu+\pi=1$. The notion of IFSs provides a flexible framework to elaborate uncertainty and vagueness. There are lots of research work done in the area of IFSs in [3-6].

There are situations where $\mu+\nu\geq 1$, unlike the cases capture in IFSs. This limitation in IFS naturally led to a construct, called Pythagorean fuzzy sets (PFSs). Pythagorean fuzzy set (PFS) proposed in [7, 8] is a new tool to deal with vagueness considering the membership grade, μ and nonmembership grade, ν satisfying the co,tions or $0\leq\nu\leq 1$, and also, it follows that $\mu^2+\nu^2+\pi^2\leq 1$, where π is the pythagorean fuzzy set index. The construct of PFSs can be used to characterize uncertain information more sufficiently and accurately than IFS. Garg [9] presented an improved score function for the ranking order of interval-valued Pythagorean fuzzy sets (IVPFSs). Based on it, a pythagorean fuzzy technique for order of preference by similarity to ideal solution (TOPSIS) method by taking the preferences of the experts in the form of interval-valued Pythagorean fuzzy decision matrices was discussed [10]. Other explorations of the theory of PFSs can be found in [11-12]. Obviously, PFS is more capable than IFS to model the vagueness in the practical problem.

The notions of fuzzy subnear-ring and ideal were first introduced by Abou-Zaid in [13]. The notion of fuzzy ideals of near rings with interval valued membership functions introduced by Davvaz [14] in 2001. Kim and Jun [15] in their paper entitled "Normal fuzzy R-subgroups in nearrings" introduced the concept of a normal fuzzy R-subgroup in near-rings and explored some related properties. In [16], Kuncham et al., introduced fuzzy prime ideal of near-rings.

In[17] Biswas introduced the concept of anti-fuzzy subgroups of groups, Kim and Jun studied the notion of anti-fuzzy R-subgroups of near-ring in [18], and Kim et al. studied the notion of anti-fuzzy ideals in near-rings and introduced the concept of generalized anti-fuzzy bi-ideals in ordered semigroups.

The rest of the paper organized as follows. In Section 2, the preliminaries and some definitions are given and present some algebraic structures of Pythagorean fuzzy sets. In Section 3, we studied the definition of the pythagorean fuzzy ideal of a near ring and discussed some important properties of pythagorean fuzzy ideals of a near ring. Finally, a conclusion is made in Section 4.

2 Preliminaries and Definitions

In this section, we recall the related concepts to the fuzzy sets, the intuitionistic fuzzy sets and the pythagorean fuzzy sets as the definition of intuitionistic fuzzy set, pythagorean fuzzy set. The definition of upper and lower cut on a fuzzy set is represented. We also give an analysis of this concept when applied to the pythagorean fuzzy set.

Definition 1. A fuzzy set A in a universal set X is defined as $A=\{\langle x,\mu_A(x)\rangle \mid x\in X\}$ where $\mu_A: X\to[0,1]$ is a mapping called the membership function of the fuzzy set A.

The complement of μ denoted by $\overline{\mu}$, is the fuzzy set in X given by $\overline{\mu}(x)=1-\mu(x)$ for all $x\in X$.

Definition 2. For any $t\in[0,1]$ and a fuzzy set μ in a non-empty set X, the set $U(\mu;t)=\{x\in X\mid \mu(x)\geq t\}$ is called an upper t-level cut of μ, and the set $L(\mu;t)=\{x\in X\mid \mu(x)\leq t\}$ is called a lower t-level cut of μ.

Definition 3. Let X be a fixed set. An intuitionistic fuzzy set (IFS) I in X is an expression having the form

$$I = \{\langle x, \mu_I(x), \nu_I(x)\rangle : x \in X\},$$

where the functions $\mu_I(x)$ and $\nu_I(x)$ are the degree of membership and the degree of non-membership of the element $x \in X$ respectively. Also $\mu_I : X \to [0,1]$, $\nu_I : X \to [0,1]$ and $0 \le \mu_I(x) + \nu_I(x) \le 1$, for all $x \in X$.

The degree of indeterminacy $\pi_I(x) = 1 - \mu_I(x) - \nu_I(x)$.

In practice, may be for some reason, the condition $0 \le \mu(x) + \nu(x) \le 1$, is not true. For instance, $0.4 + 0.7 = 1.1 > 1$ but $0.4^2 + 0.7^2 < 1$, or $0.5 + 0.7 = 1.2 > 1$ but $0.5^2 + 0.7^2 < 1$. To overcome this situation, in 2013 Yager[24] introduced the concept of the pythagorean fuzzy set.

Definition 4. A PFS P in a finite universe o discourse X is given by

$$P = \{\langle x, \mu_P(x), \nu_P(x)\rangle \mid x \in X\},$$

where $\mu_P(x) : X \to [0,1]$ denotes the degree of membership and $\nu_P(x) : X \to [0,1]$ denotes the degree of non-membership of the element $x \in X$ to the set A respectively with the condition that $0 \le (\mu_P(x))^2 + (\nu_P(x))^2 \le 1$.

The degree of indeterminacy $\pi_P(x) = \sqrt{1 - (\mu_P(x))^2 - (\nu_P(x))^2}$.

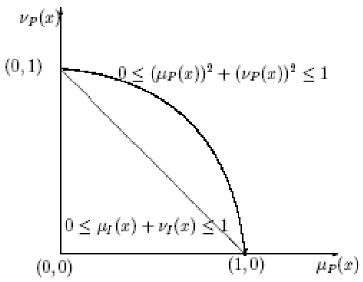

Fig. 1 Pythagorean fuzzy set

2.1. Some operations on Pythagorean Fuzzy Numbers

Given three PFNs $\alpha = \langle \mu, \nu \rangle$, $\alpha_1 = \langle \mu_1, \nu_1 \rangle$ and $\alpha_2 = \langle \mu_2, \nu_2 \rangle$. Yager defined the basic operations which can be defined as follows:

(1)$\overline{\alpha} = \langle \nu, \mu \rangle$

(2)$\alpha_1 \vee \alpha_2 = \langle \max\{\mu_1, \mu_2\}, \min \nu_1, \nu_2 \rangle$

(3) $\alpha_1 \wedge \alpha_2 = \langle \min\{\mu_1, \mu_2\}, \max \nu_1, \nu_2 \rangle$

(4)$\alpha_1 \oplus \alpha_2 = \langle \sqrt{\mu_1^2 + \mu_2^2 - \mu_1^2 \mu_2^2}, \nu_1 \nu_2 \rangle$

(5)$\alpha_1 \otimes \alpha_2 = \langle \mu_1 \mu_2, \sqrt{\nu_1^2 + \nu_2^2 - \nu_1^2 \nu_2^2} \rangle$

(6)$\lambda \cdot \alpha = \langle \sqrt{1 - (1 - \mu^2)^\lambda}, \nu^\lambda \rangle, \lambda > 0.$

(7)$\alpha^\lambda = \langle \mu^\lambda, \sqrt{1 - (1 - \nu^2)^\lambda} \rangle, \lambda > 0.$

Definition 5. Let $\alpha_1 = \langle \mu_1, \nu_1 \rangle$ and $\alpha_2 = \langle \mu_2, \nu_2 \rangle$ be two PFNs; $S(\alpha_1) = \mu_1^2 - \nu_1^2$ and $S(\alpha_2) = \mu_2^2 - \nu_2^2$ be their score functions; $H(\alpha_1) = \mu_1^2 + \nu_1^2$ and $H(\alpha_2) = \mu_2^2 + \nu_2^2$ be the accuracy degrees of α_1 and α_2, then Yager and Abbasov [24] defined the following:

1) *If* $S(\alpha_1) < S(\alpha_2)$ *then* α_1 *is smaller than* α_2, *that is* $\alpha_1 < \alpha_2$;

2) $If\ S(\alpha_1) > S(\alpha_2)\ then\ \alpha_1 > \alpha_2$,
3) $If\ S(\alpha_1) = S(\alpha_2)\ then$
$if\ H(\alpha_1) < H(\alpha_2)\ then\ \alpha_1 < \alpha_2$
$if\ H(\alpha_1) > H(\alpha_2)\ then\ \alpha_1 > \alpha_2$
$if\ H(\alpha_1) = H(\alpha_2)$ *then* α_1 *and* α_2 *represent the same information, that is* $\alpha_1 = \alpha_2$.

3. Pythagorean Fuzzy Ideals of a Near-ring

In this section, we recall some basic definitions of near-rings.
A near ring is a non-empty set R with two binary operations $+$ and $.$ satisfying the following axioms:

(i) $(R,+)$ is a group,

(ii) $(R,.)$ is a semigroup,

(iii) $x.(y+z) = x.y + x.z$ for all x, y, $z \in R$.

Precisely speaking, it is a left near-ring because it satisfied left distributive law. We will use the word "near ring " instead of "left near ring ". Note that $x.0 = 0$ and $x(-y) = -xy$ for all $x, y \in R$ but in general $0.x \neq 0$ for some $x \in R$.
An ideal of a near ring R is a subset I of R such that

(i) $(I,+)$ is a normal subgroup of $(R,+)$

(ii) $RI \subset I$

(iii) $(x+a)y - xy \in I$ for all $a \in I$ and $x, y \in R$.

Definition 6. A fuzzy set A in a near ring R is called a fuzzy ideal of R if it satisfies following properties

$(i)\mu(x-y) \geq \min\{\mu(x), \mu(y)\}$

$(ii)\mu(y+x-y) \geq \mu(x)$

$(iii)\mu(xy) \geq \mu(y)$

$(iv)\mu((x+z)y - xy) \geq \mu(z),\ for\ all\ x, y, z \in R.$

Definition 7. A fuzzy set A in a near ring R is called a anti fuzzy ideal of R if it satisfies following properties

$(i)\mu(x-y) \leq \max\{\mu(x), \mu(y)\}$

$(ii)\mu(y+x-y) \leq \mu(x)$

$(iii)\mu(xy) \leq \mu(y)$

$(iv)\mu((x+z)y - xy) \leq \mu(z),\ for\ all\ x, y, z \in R.$

Definition 8. Let $P = (\mu_P, \nu_P)$ be a pythagorean fuzzy set in a near ring R. Then P is called a pythagorean fuzzy ideal in a near ring R if it satisfies

$(i)\mu_P(x-y) \geq \min\{\mu_P(x), \mu_P(y)\}$

$(ii)\mu_P(y+x-y) \geq \mu_P(x)$

$(iii)\mu_P(xy) \geq \mu_P(y)$

$$(iv)\mu_P((x+z)y-xy)\geq \mu_P(z)$$
$$(v)\nu_P(x-y)\leq \max\{\nu_P(x),\nu_P(y)\}$$
$$(vi)\nu_P(y+x-y)\leq \nu_P(x)$$
$$(vii)\nu_P(xy)\leq \nu_P(y)$$
$$(viii)\nu_P((x+z)y-xy)\leq \nu_P(z),\ \textit{for all}\ x,y,z\in R.$$

Example 1. Let $R=\{a,b,c,d\}$ be a set with two binary operations as follows:

$+$	a	b	c	d	$\cdot$	a	b	c	d
a	a	b	c	d	a	a	a	a	a
b	b	a	d	c	b	a	a	a	a
c	c	d	b	a	c	a	a	a	a
d	d	c	a	b	d	a	a	b	b

Then $(R,+,\cdot)$ is a near ring. Let PFS $P=(\mu_P,\nu_P)$ in R defined by $\mu_P(a)=0.8$, $\mu_P(b)=0.6$, $\mu_P(c)=\mu_P(d)=0.3$ and $\nu_P(a)=0.2$, $\nu_P(b)=0.3$, $\nu_P(c)=\nu_P(d)=0.7$.

It can be shown that pythagorean fuzzy set $P=(\mu_P,\nu_P)$ is an pythagorean ideal of R.

Theorem 1. Let $P=(\mu_P,\nu_P)$ be a pythagorean fuzzy set in a near ring R. Then P is a pythagorean fuzzy ideal of a near ring R if and only if μ_P and $\bar{\nu}_P$ are fuzzy ideals of R.

Proof. Let PFS $P=(\mu_P,\nu_P)$ is a pythagorean fuzzy ideal of a near ring R. Then μ_P is a fuzzy ideal. Now, for every $x,y\in R$, we have

$$\bar{\nu}_P(x-y)=1-\nu_P(x-y)\geq 1-\max\{\nu_P(x),\nu_P(y)\}=\min\{1-\nu_P(x),1-\nu_P(y)\}=\min\{\bar{\nu}_P(x),\bar{\nu}_P(y)\}$$
$$\bar{\nu}_P(y+x-y)=1-\nu_P(y+x-y)\geq 1-\nu_P(x)=\bar{\nu}_P(x)$$
$$\bar{\nu}_P(xy)=1-\nu_P(xy)\geq 1-\nu_P(y)=\bar{\nu}_P(y)$$
$$\bar{\nu}_P((x+z)y-xy)=1-\nu_P((x+z)y-xy)\geq 1-\nu_P(z)=\bar{\nu}_P(z)$$

Hence $\bar{\nu}_P$ is a fuzzy ideal of R.

Conversely, assume that μ_P and $\bar{\nu}_P$ are fuzzy ideals of a near ring R. For every x, y, $z\in R$ we get

$$\bar{\nu}_P(x-y)\geq \min\{\bar{\nu}_P(x),\bar{\nu}_P(y)\}$$
$$\textit{and that is},\ 1-\nu_P(x-y)\geq \min\{1-\nu_P(x),1-\nu_P(y)\}$$
$$=1-\max\{\nu_P(x),\nu_P(y)\}$$
$$\textit{that is}\ \nu_P(x-y)\leq \max\{\nu_P(x),\nu_P(y)\}.$$
$$\bar{\nu}_P(y+x-y)\geq \bar{\nu}_P(x)$$
$$i.e.,\ 1-\nu_P(y+x-y)\geq 1-\nu_P(x)$$
$$i.e.,\ \nu_P(y+x-y)\leq \nu_P(x)$$
$$\bar{\nu}_P(xy)\geq \bar{\nu}_P(y)$$
$$1-\nu_P(xy)\geq 1-\nu_P(y)$$
$$\nu_P(xy)\leq \nu_P(y)$$
$$\bar{\nu}_P((x+z)y-xy)\geq \bar{\nu}_P(z)$$

$$1-\nu_P((x+z)y-xy)\geq 1-\nu_P(z)$$
$$\nu_P((x+z)y-xy)\leq \nu_P(z).$$

Hence the pythagorean fuzzy set $P=(\mu_P,\nu_P)$ is a pythagorean fuzzy ideal of a near ring R.

Definition 9. Let PFS $P=(\mu_P,\nu_P)$. Then we define two operations on P as follows:

$$(1)\Box P=\langle \mu_P,\overline{\mu_P}\rangle, \textit{where } \overline{\mu_P}(x)=1-\mu_P(x).$$
$$(2)\Diamond P=\langle \overline{\nu_P},\nu_P\rangle, \textit{where } \overline{\nu_P}(x)=1-\nu_P(x).$$

Theorem 2. Let $P=(\mu_P,\nu_P)$ be a pythagorean fuzzy set in a near ring R. Then, P is a pythagorean fuzzy ideal of a near ring R if and only if $\Box P=(\mu_P,\bar{\mu}_P)$ and $\Diamond P=(\bar{\nu}_P,\nu_P)$ are pythagorean fuzzy ideal of a near ring R.

Proof. If PFS $P=(\mu_P,\nu_P)$ is a pythagorean fuzzy ideal of R then $\mu_P=\bar{\bar{\mu}}_P$ and $\bar{\nu}_P$ are fuzzy ideals of R, hence $\Box P=(\mu_P,\bar{\mu}_P)$ and $\Diamond P=(\bar{\nu}_P,\nu_P)$ are pythagorean fuzzy ideal of R.

Conversely, if $\Box P=(\mu_P,\bar{\mu}_P)$ and $\Diamond P=(\bar{\nu}_P,\nu_P)$ are pythagorean fuzzy ideals of R then the fuzzy sets μ_P and $\bar{\nu}_P$ are fuzzy ideals of a near ring R.

Hence PFS $P=(\mu_P,\nu_P)$ is a pythagorean fuzzy ideal of a near ring R.

Theorem 3. A pythagorean fuzzy set $P=(\mu_P,\nu_P)$ is a pythagorean fuzzy ideal of a near ring R if and only if for all $\alpha,\beta\in[0,1]$ the non-empty set $U(\mu;\alpha)$ and $L(\nu;\beta)$ are ideals of a near ring R.

Proof. Let pythagorean fuzzy set $P=(\mu_P,\nu_P)$ be a pythagorean fuzzy ideal of a near ring R. First, for any $\alpha,\beta\in[0,1]$, let $x,y\in U(\mu_P;\alpha)$, then $\mu_P(x)\geq\alpha$ and $\mu_P(y)\geq\beta$.

Hence $\mu_P(x-y)\geq\min\{\mu_P(x),\mu_P(y)\}\geq\alpha$ and so $x-y\in U(\mu_P;\alpha)$.

Second, for any $x\in U(\mu_P;\alpha)$ and $y\in R$, we get $\mu_P(y+x-y)\geq\mu_P(x)\geq t$, and that $y+x-y\in U(\mu_P;t)$.

Third, for any $r\in R$ and $x\in U(\mu_P;\alpha)$, we have $\mu_P(xr)\geq\mu_P(x)\geq\alpha$ and so $xr\in U(\mu_P;\alpha)$.

At last, for any $i\in U(\mu_P;\alpha)$ and $x,y\in R$, then $\mu_P((x+i)y-xy)\geq\mu_P(i)\geq t$, and that $(x+i)y-xy\in U(\mu_P;\alpha)$.

Therefore $U(\mu_P;\alpha)$ is an ideal of a near ring R.

Again, let $x,y\in L(\nu_P,\beta)$, then $\nu_P(x)\leq\beta$ and $\nu_P(y)\leq\beta$.

Hence $\nu_P(x-y)\leq\max\{\nu_P(x),\nu_P(y)\}\leq\beta$, and so $x,y\in L(\nu_P,\beta)$.

Secondly, for any $x\in L(\nu_P,\beta)$ and $y\in R$, we get $\nu_P(y+x-y)\leq\nu_P(x)\leq\beta$, and that $y+x-y\in L(\nu_P,\beta)$.

Moreover, for any $r\in R$ and $x\in L(\nu_P,\beta)$ we have $\nu_P(xr)\leq\nu_P(x)\leq\beta$, and so $xr\in L(\nu_P,\beta)$.

Finally, for any $i\in L(\nu_P,\beta)$ and $x,y\in R$, we have $\nu_P((x+i)y-xy)\leq\nu_A(i)\leq\beta$, and that $(x+i)y-xy\in L(\nu_P,\beta)$, and therefore $L(\nu_P,\beta)$ is an ideal of a near ring R.

Definition 10. A map f from a near ring R into a near ring S is called homomorphism if $f(x+y)=f(x)+f(y)$ and $f(xy)=f(x)f(y)$ for all x, $y\in R$.

Let $f:R\to S$ be a homomorphism of near rings. For any PFS $P=(\mu_P,\nu_P)$ if S we define PFS $P^f=(\mu_P^f,\nu_P^f)$ in R by $\mu_P^f(x)=\mu_P(f(x))$, $\nu_P^f(x)=\nu_P(f(x))$ for all $x\in R$.

Theorem 4. Let $f:R\to S$ be a homomorphism of near ring. If a PFS $P=(\mu_P,\nu_P)$ in S is a pythagorean fuzzy ideal of S, then PFS $P^f=(\mu_P^f,\nu_P^f)$ in R is a pythagorean fuzzy ideal of R

Proof. For any $x,y,z\in R$,

$$(i)\ \mu_P^f(x-y)=\mu_P(f(x)-f(y))$$
$$\geq \min\{\mu_P(f(x)),\mu_P(f(y))\}$$
$$=\min\{\mu_P^f((x)),\mu_P^f((y))\}.$$

$$(ii)\ \mu_P^f(y+x-y)=\mu_P(f(y+x-y))$$
$$=\mu_P(f(y)+f(x)-f(y))$$
$$\geq \mu_P(f(x))=\mu_P^f((x)).$$

$$(iii)\ \mu_P^f(xy)=\mu_P(f(xy))=\mu_P(f(x)f(y))$$
$$\geq \mu_P(f(x))=\mu_P^f(x)\ and$$

$$(iv)\ \mu_P^f((x+z)y-xy)=\mu_P(f((x+z)y-xy))$$
$$=\mu_P((f(x)+f(z))f(y)-f(x)f(y))$$
$$\geq \mu_P(f(z))=\mu_P^f((z)).$$

$$(v)\ \nu_P^f(x-y)=\nu_P(f(x)-f(y))$$
$$\leq \max\{\nu_P(f(x)),\nu_P(f(y))\}$$
$$=\max\{\nu_P^f((x)),\nu_P^f((y))\}.$$

$$(vi)\ \nu_P^f(y+x-y)=\nu_P(f(y+x-y))$$
$$=\nu_P(f(y)+f(x)-f(y))$$
$$\leq \nu_P(f(x))=\nu_P^f((x)).$$

$$(vii)\ \nu_P^f(xy)=\nu_P(f(xy))=\nu_P(f(x)f(y))$$
$$\leq \mu_P(f(x))=\nu_P^f(x)\ and$$

$$(viii)\ \nu_P^f((x+z)y-xy)=\nu_P(f((x+z)y-xy))$$
$$=\nu_P((f(x)+f(z))f(y)-f(x)f(y))$$
$$\leq \nu_P(f(z))=\nu_P^f((z)).$$

Hence $P=(\mu_P,\nu_P)$ is a pythagorean fuzzy ideal of a near ring R.

Theorem 5. Let $f:R\to S$ be an epimorphism of near-rings and let PFS $P=(\mu_P,\nu_P)$ in S. If PFS $P^f=(\mu_P^f,\nu_P^f)$ is a pythagorean fuzzy ideal of R, then PFS $P=(\mu_P,\nu_P)$ is a pythagorean fuzzy ideal of S.

Proof. Let $x,y,z\in S$, then there exist $a,b,c\in R$ such that $f(a)=x$, $f(b)=y$ and $f(c)=z$.

$$(i)\ \mu_P(x-y)=\mu_P(f(a)-f(b))=\mu_P(f(a-b))=\mu_P^f(a-b)\geq \min\{\mu_P^f(a),\mu_P^f(b)\}$$
$$=\min\{\mu_P^f(f(a)),\mu_P^f(f(b))\}=\min\{\mu_P^f(x),\mu_P^f(y)\}$$

$$(ii)\ \mu_P(y+x-y)=\mu_P(f(b)+f(a)-f(b))=\mu_P(f(b+a-b))$$
$$=\mu_P^f(b+a-b)\geq\mu_P^f(a)=\mu_P(f(a))=\mu_P(x).$$
$$(iii)\ \mu_P(xy)=\mu_P(f(a)f(b))=\mu_P(f(ab))=\mu_P^f(ab)\geq\mu_P^f(a)$$
$$=\mu_P(f(a))=\mu_P(x).$$
$$(iv)\ \mu_P((x+z)y-xy)=\mu_P((f(a)+f(c))f(b)-f(a)f(b))=\mu_P(f((a+c)b-ab))$$
$$=\mu_P^f((a+c)b-ab)\geq\mu_P^f(c)=\mu_P(f(c))=\mu_P(z).$$

Moreover,

$$(v)\ \nu_P(x-y)=\nu_P(f(a)-f(b))=\nu_P(f(a-b))=\nu_P^f(a-b)\leq\max\{\nu_P^f(a),\nu_P^f(b)\}$$
$$=\max\{\nu_P^f(f(a)),\nu_P^f(f(b))\}=\min\{\nu_P^f(x),\nu_P^f(y)\}.$$
$$(vi)\ \nu_P(y+x-y)=\nu_P(f(b)+f(a)-f(b))=\nu_P(f(b+a-b))$$
$$=\nu_P^f(b+a-b)\leq\nu_P^f(a)=\nu_P(f(a))=\nu_P(x).$$
$$(vii)\ \nu_P(xy)=\nu_P(f(a)f(b))=\nu_P(f(ab))$$
$$=\nu_P^f(ab)\leq\nu_P^f(a)=\nu_P(f(a))=\nu_P(x).$$
$$(viii)\ \nu_P((x+z)y-xy)=\nu_P((f(a)+f(c))f(b)-f(a)f(b))$$
$$=\nu_P(f((a+c)b-ab))=\nu_P^f((a+c)b-ab)$$
$$\leq\nu_P^f(c)=\nu_P(f(c))=\nu_P(z).$$

Hence the pythagorean fuzzy set $P=(\mu_P,\nu_P)$ is a pythagorean fuzzy ideal of S.

4. Conclusion

In this study the structure of a fuzzy algebraic system, we notice that the Pythagorean fuzzy ideal of near ring with special properties always plays an important role. In this paper, by means of a kind of a new idea, we defined Pythagorean fuzzy ideal of a near-ring and investigated some of its related properties. In particular, we defined concept cut set of Pythagorean fuzzy set and proved that cut set form a Pythagorean fuzzy ideal of a near ring. Also, represent the homomorphism of the pythagorean fuzzy ideal of near rings and prove some important properties. We hope that the research along this direction can be continued, and in fact, some results in this paper have already constituted a platform for further discussion concerning the future development of near-rings. In our future study of fuzzy structure of near-rings, may be the following topics: describe soft near-rings and its applications; establish an (∈; ∈∨ q)-fuzzy spectrum of near-rings.

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