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Inverse data envelopment analysis for merging two-stage network systems

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Abstract The inverse data envelopment analysis (InvDEA) technique is an applicable method in order to estimate the input/output levels of decision-making units (DMUs) to preserve predetermined technical efficiency scores. In the managerial atmosphere, the decision maker (DM) aims to merge two or more units and needs to know the input/output levels of the merged unit, while the efficiency score of the new unit is set, however, in some cases, the units have two-stage network structures. The main purpose of this paper is merging DMUs with two-stage network structures. To reach this goal, in this paper, an InvDEA method is presented in order to estimate inputs and the intermediate products of two-stage DMUs, to achieve the different predetermined efficiency scores which have been set by the DM.

Keyword: Inverse DEA, Network DEA, Two-Stage Systems, Merge and Aggregation.

1 Introduction

Data envelopment analysis which was proposed by Charnes, Cooper and Rhoades [1] is a useful technique to assess the efficiency score of homogeneous DMUs with multiple inputs and outputs. From another point of view, the InvDEA method proposed by Wie et al. [2] aims to answer the following questions:

1. If among a group of comparable DMUs, the output levels of a certain unit are increased, how many more inputs should be provided for the unit, in order that, the efficiency score of the DMU remains unchanged.

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2. If among a group of comparable DMUs, we increase certain input levels of a particular DMU and assume that the unit keeps its current efficiency score, how many more outputs could the unit produce?

In some cases, a number of units do not get the favorable efficiency score from the DM's point of view and do not reach to the desirable performance, expected by the DM. So, the DM decides to merge two or more units, so as to achieve the predetermined efficiency score. To attain this goal, there are some papers, which use the concept of InvDEA to merge DMUs. For example, [3] applied a bootstrapped DEA-based model to investigate the existence of operating efficiency gains resulting from the potential Greek banks M&As. [4] suggested a new InvDEA method for merging banks. In [4], the authors assume that the DM of the particular banking unit willing to merge with another banking unit needs to decide about the input/output levels if an efficiency target for the new banking unit is set. Moreover, a new InvDEA model for the target setting of a merger, in the presence of negative data introduced in [5]. Then, [6] investigated the problem of merging units in the presence of negative data. Moreover, [7] deal with DEA-R models in the presence of negative ratio data by proposing an InvDEA model for merger analysis that can deal with negative data. [8] developed the concept of M&A and firms restricting and presented the concept of generalized restricting using InvDEA. In addition, [9] defined minor and major consolidation concepts. They proposed a novel method to anticipate whether a merger in a market is generating a major or minor consolidation using the InvDEA method. Besides, [10] suggested a novel method to deal with target setting in mergers, by utilizing goal programming and InvDEA approaches. Then, [11] expanded the application of InvDEA in a merger by introducing a flexible target setting, which allows the DM to favor specific input in the target setting. Furthermore, [12] introduced an InvDEA, based on a cost efficiency model, for estimating the potential gains from mergers. They showed that the proposed InvDEA cost efficiency model can reveal more merger gains, than the InvDEA technical efficiency model. In addition, [13] proposed a new form of an InvDEA model, which considers the income for planning and budget, for the financing and budgeting of constraints. In the proposed model, both, the input and output levels are variable to meet the income (or budget) constraints. [14] developed two-stage InvDEA models to highlight the potential financial gains to improving efficiency in Merger and acquisition (M&A)s. The proposed two-stage InvDEA models are used to estimate potential gains from bank mergers for US commercial banks. Moreover, [15] proposed a method based on the common set of weights for studying multiple scenarios of M&As. The proposed approach allows DMs to enter their preferences within the merger analysis. As the global warming is the crucial issue in the world, there are some studies to decrease and control the greenhouse (GH) gases. For example, [16] examined the potential of M&As to energy use optimization to pairwise consolidations tomato GH farms. In addition, [17] presented a new application of InvDEA for M&A in the agricultural sector, so that the impacts of potential mergers of GH farms are investigated on the management of scarce resources.

The aforementioned studies treat the DMUs as a black-box and ignore the internal structure of the units. Therefore, this paper incorporates the concept of the InvDEA and network DEA, in order to merge DMUs with two-stage network structure and estimate the input levels and the new intermediate products, to achieve a predetermined efficiency score by the DM. Furthermore, an empirical example is given, to show the competence, of the presented method.

The rest of this paper is outlined as follows: In section 2, the concepts of DEA, InvDEA, a basic two-stage network system and merging DMUs using InvDEA are presented. Next, the

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InvDEA model for merging two-stage network systems is proposed in section 3. Finally, section 4 provides an application for the proposed InvDEA model.

2 Preliminaries

In this section, the concepts of DEA, InvDEA, merging DMUs using InvDEA and basic two-stage networks are presented.

2.1 DEA

DEA is a non-parametric method, based on mathematical programming, in order to evaluate the efficiency score of a set of homogeneous DMUs with multiple inputs and multiple outputs; which was proposed by Charnes, Cooper and Rhoades [1]. Let us assume that, there are n DMUs $(DMU_j, j=1,....,n)$ to evaluate; and also presume that each DMU_j $(j=1,....,n)$ consumes m inputs $(x_{ij}, i=1,...,m)$ to produce s outputs$(y_{rj}, r=1,...,s)$. The DMU under evaluation is called DMU_o. The input-oriented model to evaluate the efficiency value of DMU_o which is called the CCR model and was proposed by Charnes, Cooper and Rhoades [1] is:

$$
\begin{aligned}
\theta_o^* = \min \quad & \theta_o \\
s.t. \quad & \sum_{j=1}^{n} \lambda_j x_{ij} \le \theta_o x_{io}, \quad i=1,...,m \\
& \sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{ro}, \quad r=1,...,s \\
& \lambda_j \ge 0, \quad j=1,...,n.
\end{aligned}
\tag{1}
$$

In the optimal solution of model (1), if $\theta_o^*=1$, DMU_o is called CCR efficient. Otherwise DMU_o is inefficient.

2.2 Inverse DEA

Classical DEA models evaluate the performance of DMUs and assess the efficiency score of units. But in some cases, the efficiency score is known and the DM aims to estimate the input (output) levels of units after the output (input) revision. Wie et al. [2] proposed the InvDEA concept to answer this question: If among a group of comparable DMUs, the output (input) levels of DMUs are increased, how much more inputs (outputs) are required (produced) in order that, the efficiency score of units stay unchanged? Assume that the output levels of DMU_o are increased from y_o to $\beta_o = y_o + \Delta y_o$ $(\Delta y_o \ge 0\ ,\ \Delta y_o \ne 0)$. We are going to estimate the input levels $\alpha_o = x_o + \Delta x_o$ so that, the efficiency score of DMU_o would still be θ_o^*, which is obtained from model (1) by solving the following model:

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$$\begin{aligned}
&\min \ \left(\alpha_1,\alpha_2,....,\alpha_m\right)\\
&s.t. \ \sum_{j=1}^{n}\lambda_j x_{ij} \le \theta_o^* \alpha_i \ , \ i=1,...,m\\
&\sum_{j=1}^{n}\lambda_j y_{rj} \ge \beta_{ro} \ , \ r=1,...,s\\
&\lambda_j \ge 0 \ , \ j=1,...,n\\
&\alpha_i \ge 0 \ , \ i=1,...,m,
\end{aligned} \tag{2}$$

In the above model, all $x_{ij}\ (i=1,...,m)$, $y_{rj}\ (r=1,...,s)$ and $\beta_{ro}\ (r=1,...,s)$ are given and we need to obtain $\alpha_i\ (i=1,...,\mathrm{m})$s.

2.3 Basic Two-Stage Network Systems

In the basic two-stage network, where all the inputs $x_{ij}\ (i=1,...,m)$ are supplied externally and are consumed by the first stage, to produce the intermediate products $z_{gj}\ (g=1,...,h)$ and for the second stage to produce the final outputs $y_{rj}\ (r=1,...,s)$ [18]. The first stage does not produce final outputs and the second stage does not consume exogenous inputs. The structure of the basic two-stage system is depicted in Figure 1.

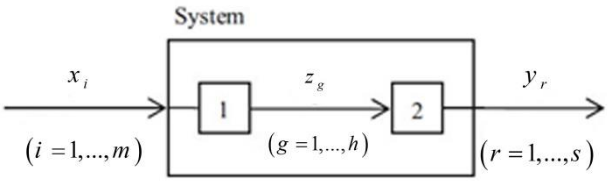

Fig.1. Structure of the basic two-stage network

There are some perspectives to evaluate the efficiency score of network systems [19] Based on the structure of the basic two-stage system shown in Figure1, the input-oriented model proposed by Kao and Hwang [19] under constant returns to scale (CRS) in a multiplier form is:

$$\begin{aligned}
&E_o^{input} = \max \ \sum_{r=1}^{s} u_r y_{ro}\\
&s.t. \quad \sum_{i=1}^{m} v_i x_{io} = 1\\
&\text{system constraints:}\\
&\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0, \quad j=1,...,n\\
&\text{division constraints:}\\
&\sum_{g=1}^{h} w_g z_{gj} - \sum_{i=1}^{m} v_i x_{ij} \le 0, \quad j=1,...,n\\
&\sum_{r=1}^{s} u_r y_{rj} - \sum_{g=1}^{h} w_g z_{gj} \le 0, \quad j=1,...,n\\
&u_r \ge 0, \quad r=1,...,s\\
&v_i \ge 0, \quad i=1,...,m\\
&w_g \ge 0, \quad g=1,...,h.
\end{aligned} \tag{3}$$

As for optimality, the system efficiency in the input-oriented form $\left(E_o^{input}\right)$ and the stage efficiencies $(E_o^{(1)}$ and $E_o^{(2)})$, are based on constrains of model (3) and can be expressed as:

$$E_o^{input}=\frac{\sum_{r=1}^{s}u_r^* y_{ro}}{\sum_{i=1}^{m}v_i^* x_{io}},\qquad E_o^{(1)input}=\frac{\sum_{g=1}^{h}w_g^* z_{go}}{\sum_{i=1}^{m}v_i^* x_{io}}\quad\text{and}\quad E_o^{(2)input}=\frac{\sum_{r=1}^{s}u_r^* y_{ro}}{\sum_{g=1}^{h}w_g^* z_{go}}. \tag{4}$$

The system efficiency is the product of the two stage efficiencies. The dual of model (3) is as follows:

$$\begin{aligned}
&E_o^{input}=\min\ \ \theta_o\\
&s.t.\ \ \sum_{j=1}^{n}\mu_j x_{ij}\le\theta_o x_{io},\quad i=1,\dots,m\\
&\sum_{j=1}^{n}\left(\mu_j-\lambda_j\right)z_{gj}\ge 0,\quad g=1,\dots,h\\
&\sum_{j=1}^{n}\lambda_j y_{rj}\ge y_{ro},\quad r=1,\dots,s\\
&\lambda_j\ge 0,\quad \mu_j\ge 0,\quad j=1,\dots,n.
\end{aligned}\tag{5}$$

The output-oriented version of model (3) is:

$$\begin{aligned}
&E_o^{output}=\min\ \sum_{r=1}^{s}v_i x_{io}\\
&s.t.\quad \sum_{r=1}^{s}u_r y_{ro}=1\\
&\text{system constraints:}\\
&\sum_{i=1}^{m}v_i x_{ij}-\sum_{r=1}^{s}u_r y_{rj}\ge 0,\quad j=1,\dots,n\\
&\text{division constraints:}\\
&\sum_{i=1}^{m}v_i x_{ij}-\sum_{g=1}^{h}w_g z_{gj}\ge 0,\quad j=1,\dots,n\\
&\sum_{g=1}^{h}w_g z_{gj}-\sum_{r=1}^{s}u_r y_{rj}\ge 0,\quad j=1,\dots,n\\
&u_r\ge 0,\quad r=1,\dots,s\\
&v_i\ge 0,\quad i=1,\dots,m\\
&w_g\ge 0,\quad g=1,\dots,h.
\end{aligned}\tag{6}$$

The dual of model (6) is as follows:

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$$
\begin{aligned}
& E_o^{output} = \max \quad \varphi_o \\
& s.t. \quad \sum_{j=1}^{n} \mu_j x_{ij} \le x_{io}, \quad i = 1,\dots,m \\
& \sum_{j=1}^{n} \left(\mu_j - \lambda_j\right) z_{gj} \ge 0, \quad g = 1,\dots,h \\
& \sum_{j=1}^{n} \lambda_j y_{rj} \ge \varphi_o y_{ro}, \quad r = 1,\dots,s \\
& \lambda_j \ge 0, \quad \mu_j \ge 0, \quad j = 1,\dots,n,
\end{aligned} \tag{7}
$$

which evaluates the output-oriented technical efficiency score of the two-stage network system.

2.4 Merging DMUs using InvDEA

After evaluating the efficiency scores of the DMUs, it is noted that some units cannot reach the desirable performance or efficiency. So, the DM decides to merge two or more units in order to achieve the predetermined efficiency score. The DM of an organization, in particular, can decide to merge two units so that: (1) one of the units remains in the PPS, or (2) both of them are omitted from PPS and make a new unit. To reach this goal, [4] assumed that the DM decides to merge DMU_l and DMU_k $\left(k \ne l\right)$. They denoted the merged unit by DMU_M and defined the set $F = \left\{i \mid 1 \le i \le n \ , \ i \ne k, l\right\}$. F means that after merging, DMU_l and DMU_k are eliminated from PPS. Then, they proposed the following InvDEA model for merging DMU_l and DMU_k $\left(k \ne l\right)$ so as to reach the predetermined efficiency score $\left(\bar{\theta}\right)$:

$$
\begin{aligned}
& \min \quad \sum_{i=1}^{m} \left(\alpha_{ik} + \alpha_{il}\right) \\
& s.t \quad \sum_{j \in F} \lambda_j x_{ij} + \left(\alpha_{ik} + \alpha_{il}\right)\lambda_M \le \bar{\theta}\left(\alpha_{ik} + \alpha_{il}\right), \quad i = 1,\dots,m \\
& \sum_{j \in F} \lambda_j y_{rj} + \left(y_{rk} + y_{rl}\right)\lambda_M \ge \left(y_{rk} + y_{rl}\right), \quad r = 1,\dots,s \\
& \sum_{j \in F} \lambda_j + \lambda_M = 1 \\
& 0 \le \alpha_{ik} \le x_{ik}, \quad i = 1,\dots,m \\
& 0 \le \alpha_{il} \le x_{il}, \quad i = 1,\dots,m \\
& \lambda_j \ge 0, \quad j \in F \\
& \lambda_M \ge 0;
\end{aligned} \tag{8}
$$

In which, $\left(\lambda, \lambda_M, \alpha_k, \alpha_l\right)$ are the variables. As it is observed, model (8) is a MONLP. It is clear that if $\bar{\theta} < 1$, then DMU_M is inefficient. So, in optimality we have $\lambda_M^* = 0$. Moreover, if DMU_M can be efficient or $\bar{\theta} = 1$, then DMU_M within the PPS can be presented in terms of

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the other efficient units. In this case, we can still suppose that in optimality $\lambda_M^* = 0$. Therefore, model (8) can be written as the following linear programming model:

$$
\begin{aligned}
&\min \sum_{i=1}^{m}(\alpha_{ik}+\alpha_{il})\\
&s.t \quad \sum_{j\in F}\lambda_j x_{ij} \le \bar{\theta}(\alpha_{ik}+\alpha_{il}), \quad i=1,...,m\\
&\sum_{j\in F}\lambda_j y_{rj} \ge (y_{rk}+y_{rl}), \quad r=1,...,s\\
&\sum_{j\in F}\lambda_j = 1\\
&0\le \alpha_{ik} \le x_{ik}, \quad i=1,...,m\\
&0\le \alpha_{il} \le x_{il}, \quad i=1,...,m\\
&\lambda_j \ge 0, \quad j\in F.
\end{aligned}
\tag{9}
$$

Theorem 1: Model (9) is feasible.
Proof: Gattoufi et al [10]. □

3 Merging Two-Stage Network Systems Using InvDEA

The merging DMUs, has been one of the considerable concepts of organizations, from managerial point of view. The merging units, in the context of being, a reconstruction of an organization, can be a strategic mode, for incrementing the production potential of the considered units, in order to achieve the predetermined efficiency score. Moreover, since in the real world, the structure of most of the units is considered as two-stage, the method of estimating inputs, intermediate products and outputs of the merged unit is questionable. In this section, an InvDEA model is proposed in order to merge the DMUs with two-stage network structure. The key aim of the proposed model is to estimate the level of inputs and intermediate products of the merged unit, so that the unit under evaluation achieves the predetermined efficiency score. Now, let us assume that, the outputs level of DMU_l and DMU_k $(k \ne l)$ are merged together so that $y_{rk}+y_{rl}$, $(r=1,...,s)$. We want to estimate the level of the inputs of $\alpha_{ik}+\alpha_{il}$, $(i=1,...,m)$ and the level of the outputs of $\gamma_{gk}+\gamma_{gl}$, $(g=1,...,h)$ such that, DMU_M reaches its predetermined efficiency score of $(\bar{\theta})$, set by the DM. For this purpose, the following MOLP model is proposed:

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$$
\begin{aligned}
&\min \ \sum_{i=1}^{m}\left(\alpha_{ik}+\alpha_{il}\right) \\
&s.t \quad \sum_{j\in F}\mu_j x_{ij} \le \bar{\theta}\left(\alpha_{ik}+\alpha_{il}\right), \qquad i=1,\dots,m \\
&\sum_{j\in F}\mu_j z_{gj} \ge \left(\gamma_{gk}+\gamma_{gl}\right), \qquad g=1,\dots,h \\
&\sum_{j\in F}\lambda_j z_{gj} \le \left(\gamma_{gk}+\gamma_{gl}\right), \qquad g=1,\dots,h \\
&\sum_{j\in F}\lambda_j y_{rj} \ge \left(y_{rk}+y_{rl}\right), \qquad r=1,\dots,s \\
&0\le \alpha_{ik} \le x_{ik}, \qquad i=1,\dots,m \\
&0\le \alpha_{il} \le x_{il}, \qquad i=1,\dots,m \\
&\gamma_{gk}\ge 0, \qquad g=1,\dots,h \\
&\gamma_{gl}\ge 0, \qquad g=1,\dots,h \\
&\lambda_j \ge 0, \qquad j\in F \\
&\mu_j \ge 0, \qquad j\in F.
\end{aligned}
\tag{10}
$$

Theorem 2: Model (10) is feasible and bounded.

Proof: according to the constraints of $0\le\alpha_{il}\le x_{il}$, $0\le\alpha_{ik}\le x_{ik}$, $\left(i=1,\dots,m\right)$, the amounts of x_{ik} and x_{il} $\left(i=1,\dots,m\right)$ are the upper bounds of α_{ik} and α_{il} , $\left(i=1,\dots,m\right)$, respectively. So, according to the constraints of $\sum_{j\in F}\mu_j x_{ij}\le\bar{\theta}\left(\alpha_{ik}+\alpha_{il}\right)$, $\left(i=1,\dots,m\right)$ the variables of μ_j $\left(j\in F\right)$ are bounded. Moreover, according to $\sum_{j\in F}\mu_j z_{gj}\ge\left(\gamma_{gk}+\gamma_{gl}\right)$, $\left(g=1,\dots,h\right)$, $\sum_{j\in F}\mu_j z_{gj}$, $\left(g=1,\dots,h\right)$ is an upper bound for $\left(\gamma_{gk}+\gamma_{gl}\right)$, $\left(g=1,\dots,h\right)$ and since $\left(\gamma_{gk}+\gamma_{gl}\right)$, $\left(g=1,\dots,h\right)$ is bounded, according to the constraint of $\sum_{j\in F}\lambda_j z_{gj}\le\left(\gamma_{gk}+\gamma_{gl}\right)$, $\left(g=1,\dots,h\right)$, the variables of λ_j $\left(j\in F\right)$ are bounded. So, model (10) is bounded.

Now, assume that, the dual of model (10) is as follows:

$$
\begin{aligned}
&Max \ \sum_{r=1}^{s}\left(y_{rk}+y_{rl}\right)D_r-\sum_{i=1}^{m}\left(x_{ik}E_i+x_{il}F_i\right) \\
&s.t. \ \sum_{g=1}^{h}B_g z_{gj}-\sum_{i=1}^{m}A_i x_{ij}\le 0, \quad j\in F \\
&\sum_{i=1}^{m}D_r y_{rj}-\sum_{g=1}^{h}C_g z_{gj}\le 0, \quad j\in F \\
&\bar{\theta}A_i-E_i\le 1, \quad i=1,\dots,m \\
&\bar{\theta}A_i-F_i\le 1, \quad i=1,\dots,m \\
&C_g-B_g\le 0, \quad g=1,\dots,h \\
&A_i\ge 0, \quad E_i\ge 0, \quad F_i\ge 0, \quad i=1,\dots,m \\
&C_g\ge 0, \quad B_g\ge 0, \quad g=1,\dots,h \\
&D_r\ge 0, \quad r=1,\dots,s
\end{aligned}
\tag{11}
$$

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Since it has been proven that model (10) is bounded, so model (11) is feasible and we can say that
$D_r = 0 \ (r = 1,...,s)$, $C_g = 0$, $B_g = 0 \ (g = 1,...,h)$, $F_i = 0$, $E_i = 0$, $A_i = 0 \ (i = 1,...,m)$ is a feasible solution of model (11). So, model (10) is feasible. □

4 Case study

In this section, the InvDEA model for merging two-stage network systems is exemplified through the data rendered by Chen and Zhu [20] and is depicted in Table 1.

Table 1 Data set of Chen and Zhu (2004)

Banks	Fixed assets ($ billion) (x_1)	IT budget ($ billion) (x_2)	# of employees (thousand) (x_3)	Deposits ($ billion) (z_1)	Profit ($ billion) (y_1)	Fraction of loans recovered (y_2)
1	0.713	0.15	13.3	14.478	0.232	0.986
2	1.071	0.17	16.9	19.502	0.34	0.986
3	1.224	0.235	24	20.952	0.363	0.986
4	0.363	0.211	15.6	13.902	0.211	0.982
5	0.409	0.133	18.485	15.206	0.237	0.984
6	5.846	0.497	56.42	81.186	1.103	0.955
7	0.918	0.06	56.42	81.186	1.103	0.986
8	1.235	0.071	12	11.441	0.199	0.985
9	18.12	1.5	89.51	124.072	1.858	0.972
10	1.821	0.12	19.8	17.425	0.274	0.983
11	1.915	0.12	19.8	17.425	0.274	0.983
12	0.874	0.05	13.1	14.342	0.177	0.985
13	6.918	0.37	12.5	32.491	0.648	0.945
14	4.432	0.44	41.9	47.653	0.639	0.979
15	4.504	0.431	41.1	52.63	0.741	0.981
16	1.241	0.11	14.4	17.493	0.243	0.988
17	0.45	0.053	7.6	9.512	0.067	0.98
18	5.892	0.345	15.5	42.469	1.002	0.948
19	0.973	0.128	12.6	18.987	0.243	0.985
20	0.444	0.055	5.9	7.546	0.153	0.987
21	0.508	0.057	5.7	7.595	0.123	0.987
22	0.37	0.098	14.1	16.906	0.233	0.981
23	0.395	0.104	14.6	17.264	0.263	0.983
24	2.68	0.206	19.6	36.43	0.601	0.982
25	0.781	0.067	10.5	11.581	0.12	0.987
26	0.872	0.1	12.1	22.207	0.248	0.972
27	1.757	0.0106	12.7	20.67	0.253	0.988

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Let us assume that the DM decides to merge DMU_{26} and DMU_{27}. The main goal is to estimate the inputs level α^*_{iM} $(i=1,2,3)$ and intermediate product γ^*_{1M} in order to achieve the predetermined amounts of $\bar{\theta}$ which are depicted in the first column of Table 2.

Table 2 Merging of DMU_{26} and DMU_{27}

$\bar{\theta}$	$\alpha^*_{1M}=\alpha^*_{1\,26}+\alpha^*_{1\,27}$	$\alpha^*_{2M}=\alpha^*_{2\,26}+\alpha^*_{2\,27}$	$\alpha^*_{3M}=\alpha^*_{3\,26}+\alpha^*_{3\,27}$	$\gamma^*_{1M}=\gamma^*_{1\,26}+\gamma^*_{1\,27}$
0.5	0.067	0.053	0.829	0.430
0.6	0.056	0.044	0.691	0.430
0.7	0.048	0.038	0.592	0.430
0.8	0.042	0.033	0.518	0.430
0.9	0.037	0.029	0.461	0.430
1.0	0.033	0.026	0.415	0.430

In the first column of Table 2, we assume that the predetermined efficiency score of DMU_M is $\bar{\theta}=0.5$. By using model (10), we can get the minimum amount of inputs and the level of the intermediate products of the merged unit. The first column of Table 2 shows that:

$$\left(\alpha^*_{1M},\ \alpha^*_{2M},\ \alpha^*_{3M},\gamma^*_{1M}\right)=(0.067,0.053,0.829,0.430)$$

Which means that DMU_M will achieve the predetermined efficiency score by the DM, if it employs the amounts of inputs and the intermediate product vector, so as to produce the final outputs of $(y_{1M},y_{2M})=(0.501,1.960)$, which is derived from merging the outputs of DMU_{26} and DMU_{27} as demonstrated hereunder:

$$y_{1M}=y_{1\,26}+y_{1\,27}=0.248+0.253=0.501$$
$$y_{2M}=y_{2\,26}+y_{2\,27}=0.972+0.988=1.960$$

5 Conclusion

In this paper, an InvDEA model was proposed for merging DMUs with two-stage network structure. In the proposed model, the outputs of two DMUs are merged to produce a new unit. Then the levels of inputs and intermediate products of the new DMU are estimated in order to achieve the predetermined efficiency score aimed by the DM. Finally, the proposed model was applied to an empirical example.

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