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A continuous multi-segmented deviation framework for multi-choice fuzzy goal programming

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Abstract This study presents an advanced formulation of multi-choice fuzzy goal programming by introducing a continuous multi-segment deviation framework that enables a refined representation of decision-maker preferences. In contrast to conventional goal programming approaches, where deviation variables are treated uniformly and weighted in a linear manner, the proposed model decomposes each deviation into several continuous segments with distinct bounds and importance weights. This structure provides the capability to model non-uniform and asymmetric preference patterns across different deviation levels while preserving the linearity of the overall formulation. Furthermore, the model incorporates flexible fuzzy aspiration levels defined through membership functions, allowing for gradual satisfaction of goals under imprecise conditions. The proposed framework offers a disaggregated and transparent representation of deviations, enhancing the interpretability of trade-offs among competing objectives. A numerical example is developed to illustrate the applicability and effectiveness of the model, and the results indicate that the proposed structure yields solutions that better reflect nuanced preference behaviors compared to traditional formulations.

Keywords: Multi-Choice Goal Programming, Segmented Deviation Structure, Fuzzy Flexible Goals, Preference Modeling.

1 Introduction

Goal Programming (GP) is a well-established approach in multi-objective decision-making and is widely used for problems involving multiple conflicting objectives and different aspiration levels. By introducing deviation variables, GP enables the transformation of a multi-objective problem into a tractable optimization problem and has therefore gained considerable importance in operations research and multi-criteria decision-making over the past decades [1-3]. Classical reviews of the literature indicate that the development of GP has focused on improving the

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representation of decision-maker preferences, objective normalization, deviation structuring, and model flexibility [3].

One of the important developments in this area is Multi-Choice Goal Programming (MCGP), introduced by Chang [4]. In this framework, the decision-maker can specify multiple aspiration levels for each goal, thereby providing greater flexibility in representing preferences than conventional GP. Chang [5] subsequently proposed a revised formulation of MCGP that eliminates certain multiplicative relationships involving binary variables, allowing the model to be formulated as a linear programming problem and solved using standard linear programming methods. This line of research has been further extended in subsequent studies, where MCGP has been integrated with other approaches to uncertainty modeling and multi-objective decision-making [6-8].

In addition to allowing multiple aspiration levels, the representation of deviation variables plays an important role in reflecting decision-maker preferences. In conventional GP, deviation from an aspiration level is generally represented by an aggregated deviation variable with a fixed weight in the objective function. However, in some decision-making problems, the importance assigned to deviations may not be uniform across the entire deviation range, and the decision-maker may exhibit different sensitivities to different portions of the deviation. In such cases, multi-segment structures can provide a more precise representation of range-dependent preferences. Liao [9] introduced Multi-Segment Goal Programming (MSGP), a framework for incorporating different levels or segments into the goal structure. Subsequently, Liao [10] proposed a revised version of this approach to provide greater flexibility in determining the coefficients associated with different segments.

On the other hand, uncertainty in parameters and aspiration levels is a fundamental issue in many applications of GP. Fuzzy Goal Programming uses membership functions to represent the gradual degree of goal achievement, allowing decision-makers to consider a range of acceptable achievement levels rather than requiring the exact attainment of a fixed target value. In recent years, the integration of Fuzzy Goal Programming with interval-based structures has received increasing attention. For example, Hocine et al. [11] proposed a fuzzy goal programming model with interval goals, providing a more flexible representation of decision-maker preferences under uncertainty. Similarly, Kouaissah and Hocine [12] developed an interval fuzzy goal programming approach for optimizing renewable energy portfolios.

More recent studies further demonstrate that integrating goal programming with fuzzy, stochastic, and multi-choice structures remains an active research area in operations research. For instance, Deliktaş et al. [7] proposed a multi-choice conic goal programming approach based on Fermatean fuzzy and stochastic concepts for the sustainable management of end-of-life building supply chains. Similarly, Mardani Najafabadi et al. [13] developed a novel interval meta-goal programming model for sustainable planning of the water–land nexus in agricultural systems. These studies demonstrate that flexibility in aspiration levels and in the representation of uncertainty can play an important role in enhancing the applicability of GP models to complex decision-making problems.

Recent applications of fuzzy goal programming to real-world problems further highlight the importance of this research area. For example, Bakhtavar et al. [14] developed a fuzzy weighted multi-objective programming model for sustainable mine tailings management and investigated the effects of different objective priorities through weighting scenarios and sensitivity analysis. Similarly, Guellil et al. [15] employed fuzzy goal programming to simultaneously address economic, energy, and environmental objectives in workforce planning. These studies demonstrate that fuzzy structures and flexible weighting schemes can be effective in modeling real-world problems characterized by conflicting objectives and uncertainty.

Despite these advances, a specific research gap remains evident. Many existing models, whether developed within the framework of multi-choice GP or within fuzzy and interval formulations, represent deviations from goals using an aggregated variable or a relatively simple penalty structure. Consequently, their ability to explicitly represent changes in the importance assigned to deviations across different portions of the deviation range remains limited. Although multi-segment structures have been investigated in the GP literature, the integration of a continuous and adjustable structure for deviation variables with multi-choice fuzzy goal programming, with the explicit aim of representing different decision-maker sensitivities across deviation ranges, remains insufficiently explored and requires further development.

Accordingly, this study proposes a multi-choice fuzzy goal programming framework with a continuous multi-segment structure for deviation variables. In the proposed framework, deviation variables are divided into multiple continuous segments, with each segment defined by a specified length and weight. Consequently, the penalty or importance assigned to deviations can vary across their range, allowing the decision-maker's different sensitivities to small and large deviations to be explicitly incorporated into the model. The proposed structure is designed to increase the flexibility of preference representation while preserving the linear structure of the model, thereby allowing the resulting problem to be solved using standard mathematical programming methods. The main contributions of this study can be summarized as follows:

- Developing a continuous multi-segment mechanism for deviation variables to represent non-uniform decision-maker preferences toward different levels of deviation;
- Integrating the multi-segment deviation structure into a multi-choice fuzzy goal programming framework to combine preference flexibility with uncertainty in aspiration levels;
- Developing an adjustable structure in which segment lengths and their corresponding weights can be determined according to the decision-maker's preferences;
- Preserving the linear structure and computational tractability of the proposed model, thereby enabling its implementation using commonly available optimization software;
- Evaluating the performance of the proposed model through a numerical example and a case study in municipal solid waste management, and comparing its results with those obtained from a model without the multi-segment structure.

The remainder of this paper is organized as follows. Section 2 presents the theoretical foundations and concepts used in developing the proposed model. Section 3 develops the proposed model and the continuous multi-segment structure for the deviation variables. Section 4 is devoted to the numerical evaluation of the proposed model and includes a numerical example, sensitivity analysis, and a case study. Section 5 discusses the results in relation to previous studies, examines the contributions and limitations of the research, and provides suggestions for future research. Finally, the references and appendices containing computational details and the model solution code are provided.

2 Theoretical foundations and conceptual preliminaries

To develop a rigorous understanding of the modeling framework employed in this study, it is necessary to establish the theoretical foundations and fundamental concepts of goal programming

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and its major extensions. These include classical goal programming, multi-choice goal programming, and fuzzy goal programming with flexible aspiration levels. Each of these frameworks contributes essential elements to the modeling of multi-objective decision problems.

The purpose of this section is to present a structured and coherent overview of these foundational models, clarify their underlying assumptions, and highlight their conceptual relationships. The focus is on theoretical definitions and modeling principles rather than computational aspects or application-specific details.

2.1 Goal programming structure

Goal Programming (GP), originally introduced by Charnes and Cooper [1], is a widely used approach for solving multi-objective decision-making problems in which several, often conflicting, goals must be considered simultaneously. Unlike classical multi-objective optimization, which seeks Pareto-optimal solutions, GP transforms each objective into a soft constraint and measures the extent to which it is satisfied.
Let G_k denote the k-th goal function, and g_k its corresponding aspiration level. A basic GP model

$$\begin{gathered} min \sum_{k=1}^{K} |G_k(X) - g_k| \\ s.t. \quad x \in X \end{gathered} \tag{1}$$

can be expressed as:

$$\begin{gathered} min\ Z = \sum_{k=1}^{K} w_k^+ d_k^+ + w_k^- d_k^- \\ s.t. \begin{cases} G_k(X) - d_k^+ + d_k^- = y_k, & k = 1, \dots, K \\ x \in X, & d_k^+, d_k^- \geq 0 \end{cases} \end{gathered} \tag{2}$$

To obtain a tractable linear formulation, deviation variables are introduced:
Here, d_k^+ and d_k^- represent positive and negative deviations from the aspiration level, and w_k^+, w_k^- denote the corresponding penalty weights. The feasible set X may include deterministic, stochastic, or fuzzy constraints depending on the application context.

2.1.1 Multi-choice goal programming (MCGP)

Multi-Choice Goal Programming (MCGP), proposed by Chang [3], extends the classical GP framework by allowing multiple aspiration levels to be associated with each goal. This reflects situations in which decision-makers consider several acceptable target values rather than a single fixed level.

For each goal G_k, a set of possible aspiration levels is defined:

$$g_{k_1}, g_{k_2}, \dots, g_{k_m}$$

The model selects one of these levels and incorporates it into the goal structure.
Conceptually, the model can be written as:

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In MCGP, for each goal G_k, a set of potential aspiration levels $g_{k_1}, g_{k_2}, \dots, g_{k_m}$ is defined. The model is designed to select one of these levels, typically through the use of binary variables or equivalent mechanisms, and incorporate it into the goal constraint. The general formulation of

$$min \sum_{i=1}^{m} \left| G_k(x) - g_{k_i} \right|, \qquad i \in \{1, \dots, m\} \qquad s.t. \quad x \in F \tag{3}$$

the model is as follows:

By introducing multiple aspiration levels, MCGP enhances the flexibility and realism of goal modeling, particularly in situations where target values are not uniquely specified.

2.1.2 Fuzzy multi-choice goal programming (FMCGP)

In many decision-making problems, aspiration levels cannot be defined precisely and are instead expressed in vague or linguistic terms. Fuzzy multi-choice goal programming (FMCGP) addresses this issue by incorporating fuzzy set theory into the multi-choice framework.
In this approach, each aspiration level is represented by a fuzzy number, typically in triangular form:

$$\tilde{g}_{k_1}, \tilde{g}_{k_2}, \dots, \tilde{g}_{k_m}$$

The general form of the model is:

$$min \sum_{i=1}^{m} \left| G_k(x) - \tilde{g}_{k_i} \right|, \qquad i \in \{1, \dots, m\} \qquad s.t. \quad x \in F \tag{4}$$

Fuzzy membership functions are used to measure the degree of satisfaction of each goal, allowing for gradual rather than binary evaluation. This framework is particularly useful when dealing with uncertainty, imprecision, or qualitative preferences.

2.1.3 Goal programming with flexible goals

In many practical applications, it is not appropriate to define aspiration levels as rigid targets. Decision-makers often exhibit tolerance toward deviations, especially when goals are expressed in qualitative or approximate terms. To address this, flexible goal programming introduces the concept of gradual satisfaction through membership functions.

In this framework, each goal is associated with a membership function $\mu_k(0) \in [0,1]$, which represents the degree to which the goal is satisfied. A value of 1 indicates full satisfaction, while 0 corresponds to complete dissatisfaction.

Rather than strictly minimizing deviations, the model seeks to achieve higher levels of satisfaction by considering the relative desirability of different outcomes. This approach allows for a more realistic representation of decision-maker preferences and is particularly suitable for problems involving subjective or imprecise objectives.

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2.2 Summary of symbols and preliminary definitions

Throughout the modeling process presented in this paper, a range of symbols, variables, and parameters are employed, some of which may not be immediately familiar to readers without a strong background in goal programming and fuzzy optimization. To facilitate a clear understanding of the model structure, this subsection provides a concise summary of key symbolic definitions and recurring concepts. These descriptions are intended as a practical reference and are aligned with standard definitions commonly found in the classical literature. Following this overview, a representative multi-choice goal programming model with fuzzy goals is introduced to illustrate the symbolic formulation of the proposed structure. The corresponding deterministic equivalent of this model is also presented to clarify how fuzzy parameters are handled and transformed during the defuzzification process.

The following model illustrates a multi-choice goal programming formulation with flexible fuzzy goals:

$$\begin{aligned} & G_k(X) \succcurlyeq^F g_{k,min} \text{ and } \preccurlyeq^F g_{k,max}, && k = 1,2,\dots,K \\ & G_v(X) \succcurlyeq^F g_{v,min} \text{ and } \preccurlyeq^F g_{v,max}, && v = K+1,\dots,V \\ & \sum_{j=1}^{n} a_{ij}\, x_j \begin{bmatrix} \geq \\ = \\ \leq \end{bmatrix} b_I, && i = 1,2,\dots,m \\ & x_j \geq 0 && j = 1,2,\dots,n \end{aligned} \tag{5}$$

Subsequently, the parameters and variables involved are defined according to the conventions typically adopted in goal programming models:

$G_k(X)$: Fuzzy flexible goals of the first category, in which the decision-maker prioritizes reaching the minimum value of a continuous aspiration level.

$G_v(X)$: Fuzzy flexible goals of the second category, for which achieving the maximum value of the aspiration level is of greater importance to the decision-maker.

$[g_{k,min}, g_{k,max}]$: The continuous fuzzy aspiration interval for goals in the first category, representing the acceptable range specified by the decision-maker.

$[g_{v,min}, g_{v,max}]$: The continuous fuzzy aspiration interval for goals in the second category, indicating the upper and lower bounds of the desired satisfaction region.

$P_{k,max}$: Deviation from the upper bound of the aspiration interval for first-category goals, measuring the extent of overshooting the acceptable maximum.

$P_{k,min}$: Deviation from the lower bound of the aspiration interval for first-category goals, indicating the degree of shortfall from the desired minimum level.

$P_{v,max}$: Deviation from the upper bound for second-category goals, reflecting the sensitivity to failure in reaching the maximum desired level.

$P_{v,min}$: Deviation from the lower bound for second-category goals, capturing the decision-maker's aversion to underachievement relative to the lower threshold.

λ_k: Minimum membership degree assigned to fuzzy flexible goals of the first category, where $0 < \lambda_k < 1$.

λ_v: Minimum membership degree assigned to fuzzy flexible goals of the second category, where $0 < \lambda_v < 1$.

Based on the defined parameters and the conventional structure used for modeling multi-choice flexible fuzzy goals, the problem is reformulated as the following linearized goal programming

$$\begin{gathered}
min\ Z = \sum_{k=1}^{K} w_k^{+} d_k^{+} + w_k^{-} d_k^{-} + w_k^{'+} e_k^{+} + w_k^{'-} e_k^{-} + \sum_{v=K+1}^{V} w_v^{+} d_v^{+} + w_v^{-} d_v^{-} + w_v^{'+} e_v^{+} + w_v^{'-} e_v^{-} \\
G_k(X) - d_k^{+} + d_k^{-} = y_k, \quad k = 1, \dots, K \\
y_k - e_k^{+} + e_k^{-} = g_{k,min} - P_{k,min}(1-\lambda_k) \\
g_{k,min} - P_{k,min}(1-\lambda_k) \le y_k \le g_{k,max} + P_{k,max}(1-\lambda_k) \\
G_v(X) - d_v^{+} + d_v^{-} = y_v, \quad v = K+1, \dots, V \\
y_v - e_v^{+} + e_v^{-} = g_{v,max} + P_{k,max}(1-\lambda_v) \\
g_{v,min} - P_{v,min}(1-\lambda_v) \le y_v \le g_{v,max} + P_{k,max}(1-\lambda_v) \\
\sum_{j=1}^{n} a_{ij}\, x_j \begin{bmatrix} \ge \\ = \\ \le \end{bmatrix} b_i, \qquad i = 1,2, \dots, m \\
0 < \lambda_k < 1, \qquad 0 < \lambda_v < 1, \qquad X = (x_1, \dots, x_n), \qquad x_j \ge 0, \qquad j = 1, \dots, n
\end{gathered} \tag{6}$$

model.

3 . Proposed continuous segmented model

3.1 Continuous segmented representation of deviation variables

In the proposed framework, each deviation variable is decomposed into a set of continuous segments in order to capture non-uniform penalty behavior while preserving linearity.

$$d_k^{+} = \sum_{f=1}^{F} d_{kf}^{+} \tag{7}$$

For each goal k, the positive deviation variable is expressed as:

$$0 = L_{k0} < L_{k1} < \cdots < L_{kF} \tag{8}$$

The deviation domain is partitioned using predefined breakpoints:

$$\delta_{kf} = L_{kf} - L_{k(f-1)}, \qquad f = 1, \dots, F \tag{9}$$

The length of each segment is defined as:

$$0 \le d_{kf}^{+} \le \delta_{kf}, \qquad f = 1, \dots, F \tag{10}$$

Each segment variable is bounded by:

3.2 Objective Function Reformulation

The contribution of deviations to the objective function is reformulated as:

$$w_k^+ d_k^+ = \sum_{f=1}^{F} w_{kf}^+ \, d_{kf}^+ \tag{11}$$

$$\sum_{f=1}^{F} w_{kf}^+ = 1 \,, \qquad w_{k1}^+ \le w_{k2}^+ \le \cdots \le w_{kF}^+ \tag{12}$$

with:

This structure allows increasing penalties for higher deviation levels.

3.3 Sequential Activation Mechanism

To enforce a logical progression across segments, a continuous activation variable $\theta_{kf} \in [0,1]$ is introduced.

$$d_{kf}^+ \le \delta_{kf}.\theta_{kf}, \qquad \forall f = 1, \dots, F \tag{13}$$

Each segment satisfies:

$$\delta_{k(f-1)}\theta_{kf} \le d_{k(f-1)}^+ + \varepsilon\delta_{k(f-1)}, \qquad f = 2, \dots, F \tag{14}$$

Sequential activation is ensured by:

where $\varepsilon > 0$ is a small constant.

$$\delta_{kf}\theta_{kf} \le d_{kf}^+ + \varepsilon\delta_{kf}, \qquad f = 1,2, \dots, F \tag{15}$$

To maintain consistency between activation and actual deviation:

3.4 Complete Segment Structure

$$min \sum_{f=1}^{F} w_{kf}^+ \, d_{kf}^+ \tag{16}$$

The complete formulation for each goal k is given by:

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$$
\begin{gathered}
d_k^+ = \sum_{f=1}^{F} d_{kf}^+ \\
\sum_{f=1}^{F} w_{kf}^+ = 1 \\
d_{kf}^+ \leq \delta_{kf}.\theta_{kf}, \qquad \forall f = 1, \dots, F \\
\delta_{k(f-1)}\theta_{kf} \leq d_{k(f-1)}^+ + \varepsilon\delta_{k(f-1)}, \qquad f = 2, \dots, F \\
\delta_{kf}\theta_{kf} \leq d_{kf}^+ + \varepsilon\delta_{kf}, \qquad f = 1,2,3 \\
w_{k1}^+ \leq w_{k2}^+ \leq \cdots \leq w_{kF}^+ \\
w_{kf}^+, d_{kf}^+, \theta_{kf} \geq 0, \qquad \forall f = 1, \dots, F
\end{gathered} \tag{17}
$$

3.5 Key properties of the model

The proposed formulation satisfies the following properties:
Linearity: All constraints and objective components are linear.
Boundedness: Segment variables are explicitly bounded by δ_{kf}.
Continuity: The model remains continuous over the feasible region.
Solvability: The formulation can be solved using standard LP solvers.

3.6 Structural design consideration

Segmentation is applied only to primary deviation variables d_k^+ and d_k^-, as they represent the magnitude of deviation from aspiration levels and are suitable for multi-level analysis. Auxiliary deviation variables related to tolerance violations remain in their standard linear form to preserve model simplicity and computational efficiency.

4 Numerical case study and analytical evaluation of the proposed model
4.1 Numerical example

In the following multi-choice goal programming problem, the deviation from the first flexible fuzzy goal is defined within a range bounded by the maximum and minimum aspiration levels, set at 40 and 20, respectively $\left(P_{1,max} = 40, P_{1,min} = 20\right)$. Similarly, for the second goal, this range is specified as 30 and 14 $\left(P_{2,max} = 30, P_{2,min} = 14\right)$. According to the decision maker's preferences, the objective is to achieve the highest possible aspiration level (80) for the first goal and the lowest aspiration level (−60) for the second goal.

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$$
\begin{gathered}
G_1 \colon x_1 + 2x_2 \gtrsim^F 22 \;\; and \;\; \lesssim^F 80 \\
G_2 \colon x_2 - x_1 \gtrsim^F -60 \;\; and \;\; \lesssim^F -22 \\
-5x_1 + 2x_2 \leq 7 \\
-x_1 + 3x_2 \leq 30 \\
x_1 + 3x_2 \leq 90 \\
5x_1 - x_2 \leq 390 \\
-4x_1 + 79x_2 \geq 79 \\
x_1, x_2 \geq 0
\end{gathered} \tag{18}
$$

To obtain the optimal solution of the above problem, the first step is to derive its deterministic form, which is given as follows:

$$
\begin{gathered}
min\, Z = d_1^+ + d_1^- + e_1^+ + e_1^- + d_2^+ + d_2^- + e_2^+ + e_2^- \\
x_1 + 2x_2 - d_1^+ + d_1^- = y_1 \\
y_1 - e_1^+ + e_1^- = 80 + 40(1 - \alpha_1) \\
22 - 20(1 - \alpha_1) \leq y_1 \leq 80 + 40(1 - \alpha_1) \\
x_2 - x_1 - d_2^+ + d_2^- = y_1 \\
y_2 - e_2^+ + e_2^- = -60 - 14(1 - \alpha_2) \\
-60 - 14(1 - \alpha_2) \leq y_2 \leq -22 + 30(1 - \alpha_2) \\
-5x_1 + 2x_2 \leq 7 \\
-x_1 + 3x_2 \leq 30 \\
x_1 + 3x_2 \leq 90 \\
5x_1 - x_2 \leq 390 \\
-4x_1 + 79x_2 \geq 79 \\
x_1, x_2, d_1^+, d_1^-, d_2^+, d_2^-, e_1^+, e_1^-, e_2^+, e_2^- \geq 0 \\
0 \leq \alpha_1 \leq 1, \qquad 0 \leq \alpha_2 \leq 1
\end{gathered} \tag{19}
$$

By solving the above problem using LINGO software, the optimal solution is obtained as follows:

$x_1 = 36, x_2 = 22, d_1^+ = 0, d_1^- = 0, d_2^+ = 0, d_2^- = 14, e_1^+ = 0, e_1^- = 0, e_2^+ = 70.26667, e_2^- = 0$

$y_1 = 80, y_2 = 0, \alpha_1 = 1, \alpha_2 = 0.266667, G_1 = 80, G_2 = -14, Z = 84.26667$

In the subsequent analysis, the impact of the proposed structure is assessed by modeling selected goal deviations in a segmented form, depending on the direction of deviation that is of greater concern to the decision maker. Specifically, greater sensitivity is applied to deviations below the aspiration level for the first goal, and to deviations above the aspiration level for the second goal. This modeling configuration facilitates a more precise evaluation of the proposed strategy's effectiveness in comparison with commonly adopted approaches.

Based on the predefined tables for each goal, where the segment lengths and corresponding weights for each deviation component have been specified according to the decision maker's preferences, the deterministic form of the model incorporating the proposed structure is presented as follows:

Table 1 Segment parameters – deviation variables d_{1f}

Segment f	Segment Length δ_{1f}	Penalty Weight w_{kf}	Associated Value Range
1	4	0.2	[18,22]
2	6	0.3	[12,18)
3	10	0.5	[2,12)

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Table 2 Segment parameters – deviation variables d_{2f}

Segment f	Segment Length δ_{2f}	Penalty Weight w_{kf}	Associated Value Range
1	5	0.2	$[-22, -17)$
2	10	0.3	$[-17, -7)$
3	15	0.5	$[-7, +8]$

Table 3 Summary of the parameters used in the numerical example

Parameter	Value	Parameter	Value
$g_{1,min}$	22	$g_{2,min}$	-60
$g_{1,max}$	80	$g_{2,max}$	-22
$P_{1,min}$	20	$P_{2,min}$	14
$P_{1,max}$	40	$P_{2,max}$	30

For greater clarity and reproducibility of the numerical example, the main parameters associated with the aspiration levels, flexibility bounds, and segmentation structure are summarized in Table 3. The structural constraints of the problem are fully specified in Eqs. (18)–(20). Therefore, using the parameter values reported in Table 3 together with the proposed formulation, the numerical example can be fully reconstructed.

$$
\begin{gathered}
min\, Z = d_1^+ + \left(\sum_{f=1}^{3} w_{1f}^- \, d_{1f}^-\right) + e_1^+ + e_1^- + \left(\sum_{f=1}^{3} w_{2f}^+ \, d_{2f}^+\right) + d_2^- + e_2^+ + e_2^- \\
x_1 + 2x_2 - d_1^+ + d_1^- = y_1 \\
y_1 - e_1^+ + e_1^- = 80 + 40(1-\alpha_1) \\
22 - 20(1-\alpha_1) \leq y_1 \leq 80 + 40(1-\alpha_1) \\
x_2 - x_1 - d_2^+ + d_2^- = y_1 \\
y_2 - e_2^+ + e_2^- = -60 - 14(1-\alpha_2) \\
-60 - 14(1-\alpha_2) \leq y_2 \leq -22 + 30(1-\alpha_2) \\
-5x_1 + 2x_2 \leq 7 \\
-x_1 + 3x_2 \leq 30 \\
x_1 + 3x_2 \leq 90 \\
5x_1 - x_2 \leq 390 \\
-4x_1 + 79x_2 \geq 79 \\
d_1^- = \sum_{f=1}^{3} d_{1f}^- \\
d_2^+ = \sum_{f=1}^{3} d_{2f}^+ \\
d_{1f}^- \leq \delta_{1f}.\theta_{1f}, \qquad \forall f = 1,2,3 \\
d_{2f}^+ \leq \delta_{2f}.\theta_{2f}, \qquad \forall f = 1,2,3 \\
\delta_{k(f-1)}\theta_{kf} \leq d_{k(f-1)}^+ + \varepsilon\delta_{k(f-1)}, \qquad f = 2,3, \qquad k = 1,2 \\
\delta_{kf}\theta_{kf} \leq d_{kf}^+ + \varepsilon\delta_{kf}, \qquad f = 1,2,3, \qquad k = 1,2 \\
x_1, x_2, d_1^+, d_1^-, d_2^+, d_2^-, e_1^+, e_1^-, e_2^+, e_2^- \geq 0 \\
d_{1f}^-, d_{2f}^+ \geq 0, \qquad f = 1,2,3 \\
0 \leq \alpha_1, \alpha_2 \leq 1, \qquad 0 \leq \theta_{1f}, \theta_{2f} \leq 1, \qquad f = 1,2,3
\end{gathered} \tag{20}
$$

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The resulting deterministic model was implemented and solved using LINGO software. The complete code used to implement the model is provided in Appendix A. The optimal results obtained from solving the model are reported below:

$x_1 = 30, x_2 = 20, d_1^+ = 0, d_2^- = 10, e_1^+ = 0, e_1^- = 0, e_2^+ = 70.26667, e_2^- = 0$

$d_{11}^- = 4, d_{12}^- = 6, d_{13}^- = 0, d_{21}^+ = 0, d_{22}^+ = 0, d_{23}^+ = 0$

$\theta_{11} = 1, \theta_{12} = 1, \theta_{13} = 0, \theta_{21} = 0, \theta_{22} = 0, \theta_{23} = 0$

$y_1 = 80, y_2 = 0, \alpha_1 = 1, \alpha_2 = 0.2666667, G_1 = 70, G_2 = -10, Z = 82.8667$

To quantitatively evaluate the effect of incorporating the multi-segment structure for deviation variables, the results obtained from the proposed model are compared with those of the baseline model, in which the deviation variables are considered without segmentation. Since both models are solved using the same data and constraints, the comparison allows the changes in the decision-variable values, goal achievement levels, degree of flexibility, and objective function value resulting from the proposed structure to be examined. The results of this comparison are presented in Table 4.

As shown in Table 4, incorporating the multi-segment structure for the deviation variables changes the resource allocation and the resulting trade-off between the objectives. In the baseline model, the objective function value is 84.26667, whereas it decreases to 82.86667 in the proposed model. Regarding the goal achievement levels, α_1 remains equal to 1 in both models, while α_2 also remains unchanged at 0.266667. Despite these unchanged achievement levels, the decision-variable values and the resulting values of the goal functions differ between the two models. Specifically, the baseline model yields $x_1 = 36$ and $x_2 = 22$, whereas these values decrease to 30 and 20, respectively, in the proposed model. Consequently, the value of the first goal function changes from 80 to 70, while the second goal function changes from −14 to −10.

Table 4 Quantitative comparison of the baseline and proposed models

Indicator	Model without Segmentation	Proposed Model
x_1	36	30
x_2	22	20
G_1	80	70
G_2	-14	-10
α_1	1	1
α_2	0.266667	0.266667
Z	84.26667	82.86667

These results indicate that the primary effect of the multi-segment structure in this example is not merely an increase in the levels of goal achievement, but rather a change in the distribution of deviations and the ability to incorporate differentiated preferences across different deviation ranges. In the proposed model, the negative deviation associated with the first goal is divided into two segments and penalized using weights of 0.2 and 0.3, respectively. Similarly, the positive deviation associated with the second goal is controlled through the multi-segment structure and its corresponding weights. Thus, unlike the baseline model, in which the entire deviation in a given

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direction is evaluated using a uniform penalty rate, the proposed model allows different levels of deviation to be distinguished and the decision-maker's preferences to be represented progressively.

Overall, the quantitative comparison of the two models shows that the multi-segment structure can alter the trade-off between the objectives and reduce the final objective function value without changing the values of α_1 and α_2. This comparison provides a quantitative basis for examining the role of the multi-segment structure in the proposed model.

4.2 Sensitivity analysis and interpretation of results

To evaluate the robustness of the proposed model with respect to the decision-maker's preference parameters, a sensitivity analysis was conducted by varying the flexibility/aspiration levels α_1 and α_2. Since these parameters determine the degree of strictness in achieving the goals, examining the model's behavior under different values provides insights into the feasible region of the model and the changes in the final objective function value under different preference settings.

Table 5 Sensitivity analysis with respect to the aspiration parameters

α_2 \ α_1	0.1	0.2	0.266	0.3	0.4
1.0	85.2	83.8	82.87	*Infeasible*	*Infeasible*
0.9	86.8	85.4	84.476	*Infeasible*	*Infeasible*
0.8	88.4	87	86.076	*Infeasible*	*Infeasible*
0.7	90	88.6	87.676	*Infeasible*	*Infeasible*

The results presented in Table 5 indicate that the proposed model has a distinct feasibility boundary with respect to the aspiration parameters. For the values examined, the model remains feasible for α_2 values up to approximately 0.266, whereas increasing α_2 beyond this range results in an infeasible model. This behavior indicates that, beyond a certain threshold, increasing the strictness associated with the second goal becomes incompatible with the structural constraints of the problem and consequently eliminates the feasible region. Within the feasible range, decreasing the values of α_1 and α_2 results in an increase in the final objective function value Z. This behavior indicates that changes in the aspiration levels can directly affect the deviations required to satisfy the goals and, consequently, the value of the objective function. Therefore, α_1 and α_2 are not merely numerical parameters of the model; they play an important role in determining the degree of flexibility and the acceptable range of goal achievement. The results also indicate that the model is more sensitive to α_2 than to α_1 over the examined range. In particular, increasing α_2 beyond the identified threshold causes the model to become infeasible, whereas the examined changes in α_1 do not produce the same feasibility boundary. From a decision-making perspective, this sensitivity analysis can help identify an acceptable range of aspiration levels before new targets are imposed and can reduce the likelihood of selecting goals that are incompatible with the system constraints.

Figure 1 illustrates the variation in the value of Z with respect to α_2 for different values of α_1. Within the feasible region, the overall trend indicates that Z increases as α_2 decreases. In addition, the infeasible region emerges at higher values of α_2. This observation is consistent with the results

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reported in Table 4 and further indicates the existence of a threshold in the model's behavior with respect to the aspiration level associated with the second goal.

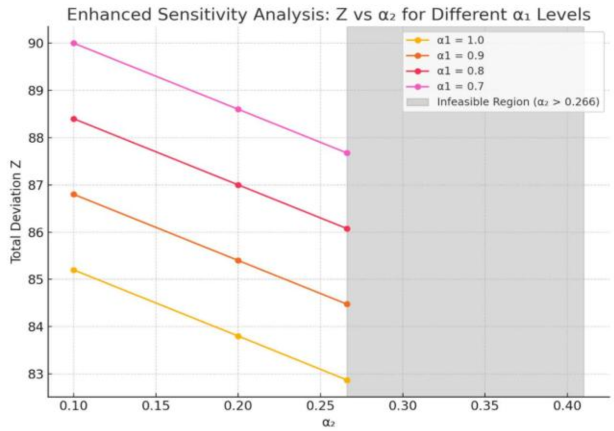

Fig. 1 Sensitivity analysis for different values of the aspiration parameters

Figure 1 presents the changes in the objective function value Z for different combinations of α_1 and α_2. The increase in Z as the aspiration levels vary, together with the emergence of an infeasible region at higher values of α_2, demonstrates the sensitivity of the proposed model to the specification of aspiration levels. Thus, the sensitivity analysis provides not only a numerical assessment of the model's behavior but also identifies a range of feasible aspiration-parameter settings that can be considered by the decision-maker.

To further evaluate the applicability of the proposed model to a problem with a more complex structure and larger dimensions, a case study in the field of municipal solid waste management was considered in addition to the numerical example presented in Section 4.1. The case study is based on a municipal solid waste allocation and processing problem and includes multiple processing facilities, multiple waste-generation sources, a multi-period planning horizon, and fuzzy and stochastic parameters. Accordingly, this case study provides an opportunity to evaluate the applicability of the proposed model to a decision-making problem with characteristics that are closer to those encountered in practical settings. Given the substantial volume of input data, parameters, constraints, and computational results associated with this case study, presenting all details in the main body of the paper would unnecessarily increase its length. Therefore, the complete problem formulation, input data, model parameters, results obtained from the proposed and baseline models, and the quantitative comparison of their results are provided in Appendix B. Together with the numerical example in Section 4.1, the case study covers two complementary aspects of model evaluation. The numerical example enables a detailed examination of the mechanism and behavior of the proposed multi-segment structure, whereas the municipal solid waste management case study demonstrates the applicability of this structure to a decision-making problem with greater dimensionality and complexity.

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5. Discussion and conclusions

5.1 Discussion

The results of this study indicate that the proposed continuous segmentation structure can enhance the flexibility of goal programming in accommodating non-uniform decision-maker preferences. In conventional approaches, deviations from aspiration levels are generally represented by aggregated variables and incorporated into the objective function using a specified penalty rate. Under such formulations, the importance assigned to deviations is assumed to be constant across the entire deviation range. However, in many decision-making problems, the importance of a deviation may change as its magnitude increases. By continuously dividing the deviation range into successive segments and assigning different weights to these segments, the proposed structure enables this behavior to be explicitly represented in the model.

The results of the numerical example demonstrated that this structure can alter the distribution of trade-offs among the objectives. In the model without segmentation, the uniform penalty assigned to deviations resulted in a considerable portion of the trade-off burden being concentrated on one objective. In contrast, in the proposed model, deviations were managed according to the predefined segments and their corresponding weights. Accordingly, the model can first utilize segments associated with lower penalties and move to higher-penalty segments only when necessary. This feature allows the intensity of deviations to be controlled progressively and avoids imposing a uniform penalty across the entire deviation range.

The comparison of the results further showed that the effect of the proposed structure is not limited to changes in the objective function value but can also be observed in the distribution of deviations among the objectives. In the model without segmentation, achieving full satisfaction of some objectives required a considerable reduction in the satisfaction level of another objective. With the segmented structure, this trade-off was distributed in a more controlled manner, allowing the satisfaction levels of the primary objectives to be maintained while reducing the extent of deterioration in the more vulnerable objective. Therefore, the proposed structure can serve as a mechanism for achieving a more balanced trade-off among conflicting objectives.

Another important feature of the proposed model is its ability to represent asymmetric decision-maker preferences. Within this framework, the importance of positive and negative deviations, or of different portions of a deviation range, does not need to be assumed identical. The decision-maker can specify the penalty structure according to the desired preferences by selecting the deviation direction, segment lengths, and corresponding weights. Consequently, the model can accommodate situations in which small deviations are considered acceptable, whereas larger deviations are penalized more heavily.

From a computational perspective, the proposed structure also offers an important advantage. Although adding multiple segments to the deviation variables increases the number of variables and constraints, the resulting formulation can still be expressed and solved as a linear programming model. Thus, increasing the flexibility of preference representation does not necessarily require a nonlinear formulation or a computationally more complex solution approach. This property facilitates the application of the proposed model to practical problems that require computational solution and the analysis of different scenarios.

The municipal solid waste management case study further illustrates the relevance of this feature in a more realistic decision-making environment. In such problems, multiple objectives, including economic costs, recycling revenues, final disposal quantities, and other operational and environmental indicators, may need to be considered simultaneously, while achieving all objectives at their desired levels may not necessarily be feasible.

Under these conditions, assigning a fixed penalty to deviations may not adequately reflect the difference between small and large deviations. The proposed structure allows the decision-maker to assign different levels of importance to different deviation ranges and, consequently, to guide the trade-offs among objectives according to the desired policy.

Overall, the findings indicate that segmenting deviation variables can provide a useful complement to the fuzzy and multi-choice structures of goal programming. Combining these features enables the model to address flexibility and uncertainty in aspiration levels while also accounting for changes in the importance assigned to deviations across their range. Consequently, the proposed framework provides greater flexibility in preference modeling than formulations in which all deviations are penalized at a constant rate.

From the perspective of comparison with existing approaches, the main distinction of the proposed model does not lie merely in increasing the number of aspiration levels or incorporating uncertainty, but rather in the way deviations from these levels are represented and controlled. While conventional approaches can model multiple aspiration levels or flexibility arising from uncertainty, the proposed framework additionally allows the importance assigned to deviations to vary across the deviation range. This capability enables the decision-maker to distinguish between small and large deviations and to adjust the penalty intensity according to the magnitude of the deviation. The comparison between the baseline and proposed models in the numerical example and the case study indicates that this structural difference can contribute to reducing the overall deviation measure and improving the achievement levels of objectives that experience greater deterioration in the model without segmentation. Accordingly, the main contribution of this study can be characterized as the development of an explicit and adjustable mechanism for the progressive allocation of deviation penalties within a multi-choice fuzzy goal programming framework.

5.2 Conclusions

This study proposed a Multi-choice fuzzy goal programming framework incorporating a continuous segmentation structure for deviation variables. The main objective was to develop a model that, in addition to the flexibility provided by fuzzy and multi-choice aspiration levels, could explicitly account for differences in the importance assigned to deviations at different levels.

In the proposed model, the deviation range associated with each goal is divided into continuous segments, with a specific length and penalty weight assigned to each segment. This enables the decision-maker to vary the penalty intensity according to the magnitude of the deviation and to represent preference structures more precisely within the model. A key advantage of this structure is its ability to represent non-uniform and asymmetric preferences while preserving the linear form of the model.

The results of the numerical example showed that incorporating the segmented structure can provide a more balanced distribution of trade-offs among the objectives compared with the model without segmentation. In the municipal solid waste management case study, the proposed model was also able to consider multiple objectives simultaneously while accounting for the problem constraints and decision-maker preferences. These findings indicate that the proposed framework

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is not limited to a theoretical example and can also be applied to practical multi-objective decision-making problems.

From a managerial perspective, the primary advantage of the proposed model is that it provides a flexible mechanism for representing decision-maker preferences. The decision-maker can adjust the trade-off structure by modifying the number of segments, the length of each segment, and its corresponding penalty weight according to the characteristics of the problem or the desired policy. Therefore, in situations where the cost or importance of a deviation changes with its magnitude, the proposed model can incorporate more detailed preference information than models based on uniform penalties.

Despite these advantages, increasing the number of segments increases the size of the model and, consequently, the number of variables and constraints required. Therefore, in large-scale applications, an appropriate balance must be maintained between the accuracy of preference representation and computational complexity. In addition, the selection of segment lengths and corresponding weights directly affects the behavior of the model. Their specification should therefore be based on decision-maker preferences, problem characteristics, and, where possible, scenario-based analyses.

Finally, the proposed framework can serve as a basis for developing more flexible goal programming models for multi-objective decision-making problems under uncertainty. Future research may focus on developing systematic procedures for determining segment weights and lengths, investigating the scalability of the proposed model for larger-scale problems, integrating the proposed structure with other forms of uncertainty, and examining its applicability in different decision-making domains.

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Appendix A. LINGO Implementation of the Numerical Model

To facilitate the reproducibility of the numerical computations, the complete LINGO code used to implement and solve the proposed model is provided in this appendix. The code includes the definitions of the model parameters, decision variables, objective function, model constraints, and the conditions associated with the segmented deviation variables. The results reported in the numerical study were obtained using this computational formulation.

```
min=d1p+0.2*d11n+0.3*d12n+0.5*d13n+e1p+e1n+0.2*d21p+0.3*d22p+0.5*d23p+d2n+e2p+e2n;
x1+2*x2-d1p+d11n+d12n+d13n=y1;
y1-e1p+e1n=80+40*(1-alpha1);
y1<=80+40*(1-alpha1);
-y1<=-22+20*(1-alpha1);
x2-x1-d21p-d22p-d23p+d2n=y2;
y2-e2p+e2n=-60-14*(1-alpha2);
y2<=-22+30*(1-alpha2);
-y2<=60+14*(1-alpha2);
-5*x1+2+x2<=7;
-1*x1+3*x2<=30;
x1+x2<=90;
5*x1-x2<=390;
4*x1-79*x2<=-79;
d11n<=4*s11n;
d12n<=6*s12n;
d13n<=10*s13n;
4*s12n<= d11n+0.001*4;
6*s13n<= d12n+0.001*6;
4*s11n<=d11n+0.001*4;
6*s12n<=d12n+0.001*6;
10*s13n<=d13n+0.001*10;
d21p<=5*s21p;
d22p<=10*s22p;
d23p<=15*s23p;
5*s22p<= d21p+0.001*5;
10*s23p<= d22p+0.001*10;
5*s21p<=d21p+0.001*5;
10*s22p<=d22p+0.001*10;
15*s23p<=d23p+0.001*15;
x1>=0;
x2>=0;
d1p>=0;
d11n>=0;
d12n>=0;
d13n>=0;
d21p>=0;
d22p>=0;
d23p>=0;
d2n>=0;
e1p>=0;
e1n>=0;
e2p>=0;
e2n>=0;
alpha1>=0;
alpha2>=0;
alpha1<=1;
alpha2<=1;
```


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```
s11n>=0;
s12n>=0;
s13n>=0;
s21p>=0;
s22p>=0;
s22p>=0;
s11n<=1;
s12n<=1;
s13n<=1;
s21p<=1;
s22p<=1;
s23p<=1;
```

Appendix B. Municipal Solid Waste Management Case Study

Consider a system in which a decision-maker is responsible for allocating municipal solid waste flows from two cities to four waste processing facilities over a 10-year planning horizon (comprising two 5-year periods indexed by (k = 1,2). The processing facilities include the following: Landfill, Incinerator, Composting facility and Recycling facility.

The capacities of the Incinerator, composting, and recycling facilities are represented by triangular fuzzy numbers.

- Incinerator: $\widetilde{TC}_1 = (280,300,320)$
- Composting facility: $\widetilde{TC}_2 = (380, 400, 420)$
- Recycling facility: $\widetilde{TC}_3 = (330,350,370)$

The capacity of the landfill is modeled as a fuzzy–random variable following a fuzzy normal distribution with the triad mean $\hat{\tilde{Z}} = (1, 2, 3) \times 10^6$ and triad variance (4, 6, 9) tons.

The residue generated at each processing facility is defined as a percentage of the incoming waste stream:

- Incinerator: 25%
- Composting facility: 10%
- Recycling facility: 8%

For each planning period, minimum diversion percentages must be satisfied based on the regulatory requirements specified in the corresponding table.

Table B.1 Diversion Rate

Facilities	Period 1 $(k = 1)$	Period 2 $(k = 2)$
Incinerator	5%	5%
Composting	8%	8%
Recycling	7%	7%

The revenue obtained from sending one ton of waste to each processing facility is given in the relevant data table.

Table B.2 Processing Revenues

Facilities	Period 1 $(k = 1)$	Period 2 $(k = 2)$
Incinerator	$\widetilde{2000}$	$\widetilde{2000}$
Composting	$\widetilde{1333}$	$\widetilde{1333}$
Recycling	$\widetilde{5111}$	$\widetilde{5111}$

Waste Generation Rates in the Two Cities: Waste generation in both cities is modeled using random variables with normal distributions:

City 1

- Period 1: mean = $\widetilde{820}$,variance = $\widetilde{20}$, probability = 0.90
- Period 2: mean = $\widetilde{840}$, variance = $\widetilde{30}$, probability = 0.85

City 2

- Period 1: mean = $\widetilde{520}$, variance = $\widetilde{12}$, probability = 0.85
- Period 2: mean =$\widetilde{550}$, variance = $\widetilde{20}$, probability = 0.90

All waste generation data are treated as random variables with normal distributions.

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Operational costs for each processing facility (USD/ton) and transportation costs (USD/ton) are specified in the input data tables.

Table B.3 Operational Costs

Facilities	Period 1 ($k = 1$)	Period2 ($k = 2$)
Landfill	$\widetilde{3134}$	$\widetilde{3447}$
Incinerator	$\widetilde{5863}$	$\widetilde{6449}$
Composting	$\widetilde{4245}$	$\widetilde{4669}$
Recycling	$\widetilde{6368}$	$\widetilde{7005}$

Transportation Costs

Table B.4 Transportation Costs

Transportation path	Period 1 ($k = 1$)	Period 2 ($k = 2$)
$city\ 1 \longrightarrow landfill$	$\widetilde{4348}$	$\widetilde{4783}$
$city\ 2 \longrightarrow landfill$	$\widetilde{4550}$	$\widetilde{5005}$
$city\ 1 \longrightarrow incinerator$	$\widetilde{4750}$	$\widetilde{5225}$
$city\ 2 \longrightarrow incinerator$	$\widetilde{4549}$	$\widetilde{5004}$
$city\ 1 \longrightarrow composting$	$\widetilde{6065}$	$\widetilde{6671}$
$city\ 2 \longrightarrow composting$	$\widetilde{6368}$	$\widetilde{7005}$
$city\ 1 \longrightarrow recyling$	$\widetilde{8288}$	$\widetilde{9117}$
$city\ 2 \longrightarrow recyling$	$\widetilde{8288}$	$\widetilde{9117}$
$incinerator \longrightarrow landfill$	$\widetilde{3040}$	$\widetilde{3344}$
$composting \longrightarrow landfill$	$\widetilde{3545}$	$\widetilde{3899}$
$recyling \longrightarrow landfill$	$\widetilde{3242}$	$\widetilde{3566}$

$$
\begin{aligned}
min\ \tilde{Z}_1 \ &= \sum_{i\in I}\sum_{j\in J}\sum_{k\in K} L_k\left(\widetilde{WTC}_{ijk} + \widetilde{OC}_{ik}\right)x_{ijk} \\
&\quad + \sum_{i\in I}\sum_{j\in J}\sum_{k\in K} L_k\left(\widetilde{RTC}_{ik} + \widetilde{OC}_{ik}\right)RF_i x_{ijk} \\
max\ \hat{Z}_2 \ &= \sum_{i\in I}\sum_{j\in J}\sum_{k\in K} L_k\ \widetilde{RE}_{ik} x_{ijk} \\
min\ \tilde{Z}_3 \ &= \sum_{i\in I}\sum_{j\in J}\sum_{k\in K} L_k\ RF_i\ x_{ijk} \\
pr&\left(\sum_{i\in I}\sum_{j\in J} L_k\left(RF_i x_{ijk}\right) \preccurlyeq \widehat{\widetilde{LC}}\right) \succcurlyeq 1-\gamma, \qquad \forall k \\
&\sum_{j\in J} x_{ijk} \preccurlyeq \widetilde{TC}_i, \quad \forall i,k \\
&\sum_{i\in I} x_{ijk} \succcurlyeq \widehat{\widetilde{W}}_{jk}, \quad \forall j,k \\
pr&\ \left(x_{ijk} \succcurlyeq \widetilde{WD}_{ik}\widehat{\widetilde{W}}_{jk}\right) \succcurlyeq 1-\beta_{jk}, \quad \forall i,j,k \\
&x_{ijk} \geq 0, \quad \forall i,j,k
\end{aligned}
\tag{B-1}
$$

Objective Functions and Numerical Results.

In the following, before presenting the final model, the deterministic version of the above multi-objective formulation (without the goal-programming and segmented structures) is first introduced. This simplified version is obtained using standard deterministic transformation techniques, in which all parameters are converted into their corresponding crisp numerical values (In the first step, the fuzzy parameters are transformed

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into their corresponding crisp values using the α-cut approach and the centroid method, while the stochastic parameters are reformulated based on the inverse quantile of the normal distribution). At this stage, the model reduces to a purely deterministic multi-objective problem, whose equations are provided in the following. In subsequent steps, this baseline formulation will be extended into the proposed final model by incorporating the goal-programming framework and the segmented structure of the deviation variables. The deterministic form of the basic multi-objective model is expressed as follows:

$$
\begin{aligned}
&min\, Z_1 = 12838.75\, x_{111} + 14122.25\, x_{112} + 12637.75\, x_{121} \\
&\qquad + 13901.25\, x_{122} + 11089\, x_{211} + 12196.8\, x_{212} \\
&\qquad + 11392 x_{221} + 12530.8\, x_{222} + 15424.8\, x_{311} \\
&\qquad + 16967.68\, x_{312} + 15424.8\, x_{321} + 16967.68 x_{322} \\
&max\, Z_2 = 1818\, x_{111} + 2000\, x_{112} + 1818\, x_{121} + 2000\, x_{122} + 1212\, x_{211} \\
&\qquad + 1333\, x_{212} + 1212 x_{221} + 1333\, x_{222} + 4646\, x_{311} \\
&\qquad + 5111\, x_{312} + 4646\, x_{321} + 5111 x_{322} \\
&min\, Z_3 = 0.25\, x_{111} + 0.25\, x_{112} + 0.25\, x_{121} + 0.25\, x_{122} + 0.1\, x_{211} \\
&\qquad + 0.1\, x_{212} + 0.1 x_{221} + 0.1\, x_{222} + 0.08\, x_{311} + 0.08\, x_{312} \\
&\qquad + 0.08\, x_{321} + 0.08 x_{322} \\
&0.25\, x_{111} + 0.25\, x_{121} + 0.10\, x_{211} + 0.10\, x_{221} + 0.08\, x_{311} + 0.08\, x_{321} \\
&\qquad \le 996860.2 \\
&0.25\, x_{112} + 0.25\, x_{122} + 0.10\, x_{212} + 0.10\, x_{222} + 0.08\, x_{312} + 0.08\, x_{322} \\
&\qquad \le 996860.2 \\
&x_{111} + x_{211} + x_{311} \le 815.300 \\
&x_{112} + x_{212} + x_{312} \le 836.233 \\
&x_{121} + x_{221} + x_{321} \le 517.380 \\
&x_{122} + x_{222} + x_{322} \le 545.242 \\
&x_{111} + x_{121} \le 316 \\
&x_{112} + x_{122} \le 316 \\
&x_{211} + x_{221} \le 416 \\
&x_{212} + x_{222} \le 416 \\
&x_{311} + x_{321} \le 366 \\
&x_{312} + x_{322} \le 366 \\
&x_{111} \ge 40.325, x_{112} \ge 40.9571, x_{121} \ge 25.433, x_{122} \ge 26.825, x_{211} \\
&\qquad \ge 64.520, x_{212} \ge 65.531, \\
&x_{221} \ge 40.693, x_{222} \ge 42.920, x_{311} \ge 56.46, x_{312} \ge 57.34, x_{321} \\
&\qquad \ge 35.606, x_{322} \ge 37.56 \\
&x_{111}, x_{112}, x_{121}, x_{122}, x_{211}, x_{212}, x_{221}, x_{222}, x_{311}, x_{312}, x_{321}, x_{322} \ge 0
\end{aligned}
\tag{B-2}
$$

After presenting the deterministic form of the basic multi-objective model, the next step introduces the innovative segmented structure for the deviation variables. In this approach, each deviation variable is partitioned into several continuous segments, where both the number of segments and the length of each segment are determined according to the decision maker's preferences. In this study, based on the input data provided by the decision maker, each deviation variable is divided into three unequal segments. The length of each segment, as well as the weight associated with that segment, is supplied to the model in the form of an input table. These weights represent the relative importance of each deviation interval, and their sum for each variable equals one.

In addition, to ensure proper comparison and aggregation of the different objectives, normalization of the deviation variables is performed according to the measurement units of each objective function. Since the first and second objectives are defined in terms of cost and revenue (unit: dollars), and the third objective is defined in terms of weight (unit: tons), their scales must be harmonized. For this purpose, the deviation values associated with the first and second objectives are multiplied by 10^6, and the deviation values associated with the third objective are multiplied by 10. This scaling ensures that all objectives are placed on a comparable basis and that differences in measurement units do not distort the optimization process. With these modifications, the segmented deterministic model is obtained. This model incorporates all components of the basic formulation

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while embedding the segmented structure of the deviation variables and the normalization of objectives. The final form of this model is presented below:

Table B.5 Objective Functions Parameters

Objective Function	Goal length	Permit tolerance bound
1	$[11924468, 16512026]$	$p_{min} = 4587558, p_{max} = 13762674$
2	$[4077750, 4957010]$	$p_{min} = 879260, p_{max} = 2637780$
3	$[115.71896, 161.72922]$	$p_{min} = 46.01026, p_{max} = 138.03078$

Table B.6 Segment Parameters

deviation variable	segment boundari	corresponding penalty weight
d_{11}^{+}	6881337	0.15
d_{12}^{+}	4128802	0.35
d_{13}^{+}	2752535	0.50
d_{21}^{-}	439613	0.15
d_{22}^{-}	263768	0.35
d_{23}^{-}	175845	0.50
d_{31}^{+}	69.01539	0.15
d_{32}^{+}	41.409234	0.35
d_{33}^{+}	27.606156	0.50

$$
\begin{aligned}
&\min\ 0.15\, H_{11}^{+} + 0.35\, H_{12}^{+} + 0.50\, H_{13}^{+} + H_{1}^{-} + C_{1}^{+} + C_{1}^{-} + H_{2}^{+} + 0.15\, H_{21}^{-} + 0.35\, H_{22}^{-} \\
&\qquad + 0.50\, H_{23}^{-} + C_{2}^{+} + C_{2}^{-} + 0.15\, H_{31}^{+} + 0.35\, H_{32}^{+} + 0.50 H_{33}^{+} + H_{3}^{-} + C_{3}^{+} \\
&\qquad + C_{3}^{-} \\
&12838.75\, x_{111} + 14122.25\, x_{112} + 12637.75\, x_{121} + 13901.25\, x_{122} + 11089\, x_{211} \\
&\qquad + 12196.8\, x_{212} + 11392 x_{221} + 12530.8\, x_{222} + 15424.8\, x_{311} \\
&\qquad + 16967.68\, x_{312} + 15424.8\, x_{321} + 16967.68 x_{322} - d_{11}^{+} - d_{12}^{+} - d_{13}^{+} \\
&\qquad + d_{1}^{-} = y_{1} \\
&y_{1} - e_{1}^{+} + e_{1}^{-} = 11924468 - 4587558(1-\alpha_{1}) \\
&11924468 - 4587558(1-\alpha_{1}) \le y_{1} \le 16512026 + 13762674(1-\alpha_{1}) \\
&1818\, x_{111} + 2000\, x_{112} + 1818\, x_{121} + 2000\, x_{122} + 1212\, x_{211} + 1333\, x_{212} + 1212 x_{221} \\
&\qquad + 1333\, x_{222} + 4646\, x_{311} + 5111\, x_{312} + 4646\, x_{321} + 5111 x_{322} - d_{2}^{+} \\
&\qquad + d_{21}^{-} + d_{22}^{-} + d_{23}^{-} = y_{2} \\
&y_{2} - e_{2}^{+} + e_{2}^{-} = 4957010 + 2637780(1-\alpha_{2}) \\
&4077750 - 879260(1-\alpha_{2}) \le y_{2} \le 4957010 + 2637780(1-\alpha_{2}) \\
&0.25\, x_{111} + 0.25\, x_{112} + 0.25\, x_{121} + 0.25\, x_{122} + 0.1\, x_{211} + 0.1\, x_{212} + 0.1 x_{221} \\
&\qquad + 0.1\, x_{222} + 0.08\, x_{311} + 0.08\, x_{312} + 0.08\, x_{321} + 0.08 x_{322} - d_{31}^{+} \\
&\qquad - d_{32}^{+} - d_{33}^{+} + d_{3}^{-} = y_{3} \\
&y_{3} - e_{3}^{+} + e_{3}^{-} = 115.71896 - 46.01026(1-\alpha_{3}) \\
&115.71896 - 46.01026(1-\alpha_{3}) \le y_{3} \le 161.72922 + 138.03078(1-\alpha_{3}) \\
&0.25\, x_{111} + 0.25\, x_{121} + 0.10\, x_{211} + 0.10\, x_{221} + 0.08\, x_{311} + 0.08\, x_{321} \le 996860.2 \\
&0.25\, x_{112} + 0.25\, x_{122} + 0.10\, x_{212} + 0.10\, x_{222} + 0.08\, x_{312} + 0.08\, x_{322} \le 996860.2 \\
&x_{111} + x_{211} + x_{311} \le 815.300 \\
&x_{112} + x_{212} + x_{312} \le 836.233 \\
&x_{121} + x_{221} + x_{321} \le 517.380 \\
&x_{122} + x_{222} + x_{322} \le 545.242 \\
&x_{111} + x_{121} \le 316 \\
&x_{112} + x_{122} \le 316 \\
&x_{211} + x_{221} \le 416 \\
&x_{212} + x_{222} \le 416
\end{aligned}
\tag{B-3}
$$

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$$x_{311} + x_{321} \le 366$$
$$x_{312} + x_{322} \le 366$$
$$x_{111} \ge 40.325, x_{112} \ge 40.9571, x_{121} \ge 25.433, x_{122} \ge 26.825, x_{211} \ge 64.520, x_{212} \ge 65.531,$$
$$x_{221} \ge 40.693, x_{222} \ge 42.920, x_{311} \ge 56.46, x_{312} \ge 57.34, x_{321} \ge 35.606, x_{322} \ge 37.56$$
$$d_{1f}^{+} \le \delta_{1f}.\theta_{1f}, \quad \forall f = 1,2,3$$
$$d_{2f}^{-} \le \delta_{2f}.\theta_{2f}, \quad \forall f = 1,2,3$$
$$d_{3f}^{+} \le \delta_{3f}.\theta_{3f}, \quad \forall f = 1,2,3$$
$$\delta_{1(f-1)}\theta_{1f} \le d_{1(f-1)}^{+} + \varepsilon\delta_{1(f-1)}, \quad f = 2,3$$
$$\delta_{1f}\theta_{1f} \le d_{1f}^{+} + \varepsilon\delta_{1f}, \quad \forall f = 1,2,3$$
$$\delta_{2(f-1)}\theta_{2f} \le d_{2(f-1)}^{-} + \varepsilon\delta_{2(f-1)}, \quad f = 2,3$$
$$\delta_{2f}\theta_{2f} \le d_{2f}^{-} + \varepsilon\delta_{2f}, \quad \forall f = 1,2,3$$
$$\delta_{3(f-1)}\theta_{3f} \le d_{3(f-1)}^{+} + \varepsilon\delta_{3(f-1)}, \quad f = 2,3$$
$$\delta_{3f}\theta_{3f} \le d_{3f}^{+} + \varepsilon\delta_{3f}, \quad \forall f = 1,2,3$$
$$\sum_{f=1}^{3} d_{1f}^{+} = 10^6 \times H_{1f}^{+}$$
$$\sum_{f=1}^{3} d_{2f}^{-} = 10^6 \times H_{2f}^{-}$$
$$\sum_{f=1}^{3} d_{3f}^{+} = 10 \times H_{3f}^{+}$$
$$d_1^{-} = 10^6 \times H_1^{-}, d_2^{+} = 10^6 \times H_2^{+}, d_3^{-} = 10 \times H_3^{-}$$
$$e_1^{+} = 10^6 \times C_1^{+}, e_1^{-} = 10^6 \times C_1^{-}, e_2^{+} = 10^6 \times C_2^{+}, e_2^{-} = 10^6 \times C_2^{-}, e_3^{+} = 10 \times C_3^{+}, e_3^{-} = 10 \times C_3^{-}$$
$$x_{111}, x_{112}, x_{121}, x_{122}, x_{211}, x_{212}, x_{221}, x_{222}, x_{311}, x_{312}, x_{321}, x_{322} \ge 0$$
$$d_{11}^{+}, d_{12}^{+}, d_{13}^{+}, d_1^{-}, e_1^{+}, e_1^{-}, d_2^{+}, d_{21}^{-}, d_{22}^{-}, d_{23}^{-}, e_2^{+}, e_2^{-}, d_{31}^{+}, d_{32}^{+}, d_{33}^{+}, d_3^{-}, e_3^{+}, e_3^{-} \ge 0$$
$$0 \le \alpha_1 \le 1, 0 \le \alpha_2 \le 1, 0 \le \alpha_3 \le 1$$
$$0 \le \theta_{1f}, \theta_{2f}, \theta_{3f} \le 1, \quad f = 1,2,3$$

The results obtained from solving the final model in LINGO are summarized below.

$$x_{111} = 40.32500, x_{112} = 40.95710, x_{121} = 25.43300, x_{122} = 26.82500, x_{211} = 64.52000, x_{212} = 65.53100, x_{221} = 40.69300, x_{222} = 42.92000, x_{311} = 330.3940, x_{312} = 328.4400, x_{321} = 35.60600, x_{322} = 37.56000$$
$$H_{11}^{+} = 4.237753, H_{12}^{+} = 0, H_{13}^{+} = 0, H_1^{-} = 0, C_1^{+} = 0, C_1^{-} = 0, H_2^{+} = 0, H_{21}^{-} = 0.4396130, H_{22}^{-} = 0.2637680, H_{23}^{-} = 0.1553714, C_2^{+} = 0, C_2^{-} = 0, H_{31}^{+} = 0, H_{32}^{+} = 0, H_{33}^{+} = 0, H_3^{-} = 0, C_3^{+} = 0, C_3^{-} = 0$$
$$\theta_{11} = 0.6158328, \theta_{12} = 0, \theta_{13} = 0, \theta_{21} = 1, \theta_{22} = 1, \theta_{23} = 0.8835703, \theta_{31} = 0, \theta_{32} = 0, \theta_{33} = 0$$
$$y_1 = 11924470, y_2 = 4957010, y_3 = 113.3114, \alpha_1 = 1, \alpha_2 = 1, \alpha_3 = 0.9476740, Z = 0.871609$$

To further evaluate the effect of the multi-segment structure of the deviation variables in the proposed model, the flexible fuzzy multi-choice goal programming model presented above was solved without applying the segmentation structure to the deviation variables. In other words, all deviation variables were considered as unified variables without being divided into multiple segments. All other components of the model, including the objective functions, constraints, parameters, and input values, were kept exactly the same as in the original formulation so that the isolated effect of removing the multi-segment structure could be clearly identified.

$$\min \quad H_1^{+} + H_1^{-} + C_1^{+} + C_1^{-} + H_2^{+} + H_2^{-} + C_2^{+} + C_2^{-} + H_3^{+} + H_3^{-} + C_3^{+} + C_3^{-}$$
$$12838.75\, x_{111} + 14122.25\, x_{112} + 12637.75\, x_{121} + 13901.25\, x_{122} + 11089\, x_{211} + 12196.8\, x_{212} + 11392 x_{221} + 12530.8\, x_{222} + 15424.8\, x_{311} + 16967.68\, x_{312} + 15424.8\, x_{321} + 16967.68 x_{322} - d_1^{+} + d_1^{-} = y_1$$

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$$y_1 - e_1^+ + e_1^- = 11924468 - 4587558(1-\alpha_1) \tag{B-4}$$
$$11924468 - 4587558(1-\alpha_1) \le y_1 \le 16512026 + 13762674(1-\alpha_1)$$
$$1818\,x_{111} + 2000\,x_{112} + 1818\,x_{121} + 2000\,x_{122} + 1212\,x_{211} + 1333\,x_{212} + 1212x_{221} + 1333\,x_{222} + 4646\,x_{311} + 5111\,x_{312} + 4646\,x_{321} + 5111x_{322} - d_2^+ + d_2^- = y_2$$
$$y_2 - e_2^+ + e_2^- = 4957010 + 2637780(1-\alpha_2)$$
$$4077750 - 879260(1-\alpha_2) \le y_2 \le 4957010 + 2637780(1-\alpha_2)$$
$$0.25\,x_{111} + 0.25\,x_{112} + 0.25\,x_{121} + 0.25\,x_{122} + 0.1\,x_{211} + 0.1\,x_{212} + 0.1x_{221} + 0.1\,x_{222} + 0.08\,x_{311} + 0.08\,x_{312} + 0.08\,x_{321} + 0.08x_{322} - d_3^+ + d_3^- = y_3$$
$$y_3 - e_3^+ + e_3^- = 115.71896 - 46.01026(1-\alpha_3)$$
$$115.71896 - 46.01026(1-\alpha_3) \le y_3 \le 161.72922 + 138.03078(1-\alpha_3)$$
$$0.25\,x_{111} + 0.25\,x_{121} + 0.10\,x_{211} + 0.10\,x_{221} + 0.08\,x_{311} + 0.08\,x_{321} \le 996860.2$$
$$0.25\,x_{112} + 0.25\,x_{122} + 0.10\,x_{212} + 0.10\,x_{222} + 0.08\,x_{312} + 0.08\,x_{322} \le 996860.2$$
$$x_{111} + x_{211} + x_{311} \le 815.300$$
$$x_{112} + x_{212} + x_{312} \le 836.233$$
$$x_{121} + x_{221} + x_{321} \le 517.380$$
$$x_{122} + x_{222} + x_{322} \le 545.242$$
$$x_{111} + x_{121} \le 316$$
$$x_{112} + x_{122} \le 316$$
$$x_{211} + x_{221} \le 416$$
$$x_{212} + x_{222} \le 416$$
$$x_{311} + x_{321} \le 366$$
$$x_{312} + x_{322} \le 366$$
$$x_{111} \ge 40.325, x_{112} \ge 40.9571, x_{121} \ge 25.433, x_{122} \ge 26.825, x_{211} \ge 64.520, x_{212} \ge 65.531,$$
$$x_{221} \ge 40.693, x_{222} \ge 42.920, x_{311} \ge 56.46, x_{312} \ge 57.34, x_{321} \ge 35.606, x_{322} \ge 37.56$$
$$d_1^+ = 10^6 \times H_1^+, d_1^- = 10^6 \times H_1^-, d_2^+ = 10^6 \times H_2^+, d_2^- = 10^6 \times H_2^-, d_3^+ = 10 \times H_3^+, d_3^- = 10 \times H_3^-,$$
$$e_1^+ = 10^6 \times C_1^+, e_1^- = 10^6 \times C_1^-, e_2^+ = 10^6 \times C_2^+, e_2^- = 10^6 \times C_2^-, e_3^+ = 10 \times C_3^+, e_3^- = 10 \times C_3^-$$
$$x_{111}, x_{112}, x_{121}, x_{122}, x_{211}, x_{212}, x_{221}, x_{222}, x_{311}, x_{312}, x_{321}, x_{322} \ge 0$$
$$d_1^+, d_1^-, e_1^+, e_1^-, d_2^+, d_2^-, e_2^+, e_2^-, d_3^+, d_3^-, e_3^+, e_3^- \ge 0$$
$$0 \le \alpha_1 \le 1, 0 \le \alpha_2 \le 1, 0 \le \alpha_3 \le 1$$

The results obtained from solving the final model in LINGO are summarized below:

$$x_{111} = 40.32500, x_{112} = 40.95710, x_{121} = 25.43300, x_{122} = 26.82500, x_{211} = 64.52000, x_{212} = 65.53100, x_{221} = 40.69300, x_{222} = 42.92000, x_{311} = 56.46000, x_{312} = 327.7106, x_{321} = 35.60600, x_{322} = 37.56000$$
$$H_1^+ = 0, H_1^- = 0, C_1^+ = 0, C_1^- = 0, H_2^+ = 0, H_2^- = 1.255918, C_2^+ = 0, C_2^- = 0.8792600, H_3^+ = 0, H_3^- = 0, C_3^+ = 0, C_3^- = 0$$
$$y_1 = 11924470, y_2 = 4077750, y_3 = 91.33835, \alpha_1 = 1, \alpha_2 = 1, \alpha_3 = 0.4701050, Z = 2.13518$$

The comparative results indicate that incorporating the multi-segment structure into the deviation variables improves the distribution of trade-offs among the objectives compared with the model without this structure. In the baseline model, while the first and second objectives achieve full satisfaction, the satisfaction level of the third objective is only 0.4701, with its value limited to 91.338 tons. In contrast, in the proposed model, while maintaining $\alpha_1 = \alpha_2 = 1$, the value of α_3 increases to 0.9477, and the value of the third objective reaches 113.311 tons. Furthermore, the overall objective function value Z decreases from 2.1352 in the baseline model to 0.8716 in the proposed model.

The results for the segmented deviation variables further indicate that deviations are allocated progressively. For the second objective, the first two segments are fully utilized, while part of the third segment is also used. In contrast, the higher-penalty segments associated with the other objectives remain inactive. These results indicate that the multi-segment structure, by assigning different weights to different deviation ranges, enables progressive control of deviations and a more balanced distribution of trade-offs among the objectives. Therefore, in this case study, the proposed model maintains the satisfaction levels of the first and second objectives while substantially reducing the deterioration in the satisfaction level of the third objective, thereby providing a more balanced performance than the baseline model.

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