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A new mathematical model for locating of relief facilities on fuzzy data using set-covering problem and data envelopment analysis

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Abstract Today, the location of rescue stations and relief facilities is very important. Responsible organizations are concerned that these facilities must be located where they cover all existing demands optimum. In this research, a new mathematical model was presented for locating of relief facilities to reduce the concerns of relevant organizations. So Set Covering Problem (SCP) and Data Envelopment Analysis (DEA) in uncertainty environment are the basic parts of this model. The Slacks-Based Model (SBM) that is one of the DEA models, is used for efficiency calculation. In this study, the introduced places are prioritized using the TOPSIS method. Prioritized places are then used as candidates points in this model with fuzzy data. Rescue & Relief Stations in the north of Fars province in Iran have been considered as a case study.

Keywords: Set-Covering Problem, Data Envelopment Analysis, Locating, Fuzzy Data, TOPSIS.

1 Introduction

Nowadays, access to emergency and rescue service facilities on roads and highways is critically vital. People expect that all emergency and rescue services will be accessible to them in the shortest possible time on the roads they travel on. Quick and easy access to medical and rescue stations has

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become a significant and critical issue for the public. Furthermore, fulfilling the demand of injured passengers and drivers in the minimum possible time is a key objective of rescue organizations. Consequently, the rescue & relief facilities location is of paramount importance. This issue significantly impacts the health of passengers and drivers traveling between cities. Providing complete demand coverage has become a crucial matter for responsible organizations, as it can directly affect the lives of passengers and drivers. Therefore, locating these facilities is among the most critical decisions for the senior managers of responsible organizations. This prompted the authors of this research to seek a comprehensive model for the facility location of these services. One well-known model in Operations Research for location planning is the Location-Allocation Model. Facility location, being one of the most important problems, addresses a wide range of applications in Supply Chain Management and Logistics, particularly in optimizing facility location to deliver products or services with customer expectations. Another prominent location model is the Set Covering Problem (SCP). The objective of the SCP is to find an optimal subset such that its members cover the entire set.

The SCP has various applications, including in emergency systems, medical and rescue facility location, defense units, military bases, etc. In this research, mathematical modeling was considered, focusing on developing a Set Covering Model. The Set Covering Model requires that the selected locations provide coverage to the entire study area and achieve an acceptable score based on indices from standards set by relevant organizations. A concern for these organizations is selecting a location with acceptable efficiency.

One of the best mathematical approaches for calculating the efficiency of Decision-Making Units (DMUs) is Data Envelopment Analysis (DEA). Given the information obtained from the relevant organization, this question arises: Is it possible to develop a novel model integrating the Set Covering Problem (used to determine candidate location points for full area coverage) and DEA (used to determine the efficiency of these points) to address the concerns of the organization while adhering to healthcare standards? Naturally, given the non-deterministic data of the modern world, this model must be planned in a non-deterministic (uncertain) environment. In this study, we aim to propose a model based on which rescue facilities can be located such that the allocated budget is also optimized. The model presented in this research is developed based on the Set Covering and DEA models, and the points considered as its inputs are determined using the TOPSIS method. The target population, as a case study, is the location planning of intercity rescue and aid stations in the northern part of Fars Province, Iran. This research utilizes the GAMS software for modeling and solving this problem.

1.1 Set-covering problem (SCP)

SCP is a classical optimization problem and one of the well-known models for facility location in operations research. The objective of SCP is to select the minimum number of subsets (or facilities) from a given collection such that every element in the universal set is covered by at least one selected subset. Facility location is one of the most critical issues in the strategic decision-making of private companies and government organizations. As a significant and central topic, facility location spans a wide range of domains, such as supply chain and logistics management, involving the optimization of facility placement to deliver expected products or services to customers. In fact, it is one of the popular models in facility location. The goal of this problem is to determine N

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service centers such that all demand points are covered with minimal deployment cost. These problems have a broad scope and nature, and each model attempts to find an optimal solution by considering specific conditions. In other words, the objective of set covering is to find a subset such that the union of its elements covers the entire set. SCP is one of the well-known models in operations research and falls under the category of integer programming. In the SCP model, the allocation of a number of facilities among several demand nodes in a given region is determined to achieve objectives such as minimizing costs, providing complete coverage, etc. SCP has various applications, including emergency systems, location of medical and relief facilities, defense units, military bases, and more.

Based on the literature review, there are four types of models as follows:

• SCP model, which aims to determine the optimal number of points that cover the entire set.

• The Maximal Covering Location Problem (MCLP) model, which locates a fixed number of points to achieve maximum coverage within the set.

• The P-Median Problem model: finding *P* points in the network to minimize the weighted distance between demand points and candidate points. This model addresses the maximum distance or time a user is separated from a facility.

• The Set Packing and Set Partitioning Problem model, used to optimize set partitions.

The first type aims to determine the minimum number of optimal points that cover the entire target set.

The second type involves locating a fixed number of points to achieve maximum coverage within the set.

The third type is used to find *P* points in the network to minimize the weighted distance between demand points and candidate points.

The fourth type is used to optimize set partitions.

Since the number of facilities in the field of relief and rescue must cover the entire area, the Set Covering Problem model has been used in this research.

1.2 Data envelopment analysis (DEA)

DEA is a non-parametric method in operations research that measures or estimates the efficiency of decision-making units. DEA can be applied to any economic activity to evaluate the efficiency of decision-making units within that domain. In DEA, regression models are sometimes employed. In microeconomics and production theory, the combinations of inputs and outputs of a firm can be considered a function. This function is known as the production function. To achieve maximum output, various combinations of input quantities and variables can be considered. This way, a technology can be identified that optimizes production for a firm or a factory.

Essentially, DEA operates based on linear programming and aims to maximize performance and identify efficiency. This method can be categorized under multi-objective linear optimization techniques. For example, DEA can be used to compare the efficiency of DMUs within a company. Additionally, DEA can be employed to measure relative efficiency across different industries or firms for comparative purposes.

DEA encompasses various models, with the foundational model known as the CCR method, named after Charnes, Cooper, and Rhodes. In facility location problems, DEA can be used to evaluate the efficiency of potential facilities, while the Set Covering Problem (SCP) ensures that

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all demand points are covered by at least one facility. For instance, in healthcare, DEA can assess the efficiency of hospitals, and SCP can guarantee that all regions are covered by at least one efficient hospital.
As previously mentioned, one of the most popular models in facility location is the Set Covering Problem, which holds particular appeal due to its practicality in real-world applications. Examples include the location of emergency facilities and medical services.
While classical DEA models like CCR (developed by Charnes, Cooper, and Rhodes) and BCC (developed by Banker, Charnes, and Cooper) operate based on radial (proportional) reduction, they have a fundamental weakness: they do not fully account for input and output slacks in the efficiency calculation. This is where the SBM model comes in.

1.2.1 Slacks-based measure (SBM)

The SBM model is a non-radial and slacks-based approach that addresses this gap.
Key features of the SBM Model:
1. Non-radial: Focuses on each input and output individually,
2. Slacks-based: Directly optimizes input excesses and output shortfalls,
3. Unit invariant: Independent of measurement units,
4. Flexible: Works with both constant and variable returns to scale.

Mathematical formulation (Input orientation):
Efficiency score $\rho^* = min\ [1 - (1/m) \sum(s_i^-/x_{i0})] \ /\ [1 + (1/s) \sum(s_r^+/y_{r0})]$ (1)
Constraints:

$$x_0 = X\lambda + s^-$$
$$y_0 = Y\lambda - s^+$$
$$\lambda \geq 0, s^- \geq 0, s^+ \geq 0$$

Where:
- ρ^* = Efficiency score (0-1)
- m = Number of inputs
- s = Number of outputs
- x_0 = Input vector of DMU under evaluation
- y_0 = Output vector of DMU under evaluation
- X = Input matrix for all DMUs
- Y = Output matrix for all DMUs
- λ = Intensity vector
- s^- = Input slacks (excesses)
- s^+ = Output slacks (shortfalls)

Interpretation:
- $\rho^* = 1$: Fully efficient (no slacks)
- $\rho^* < 1$: Inefficient (slacks present)

In Table 1, a comprehensive comparison of DEA models has been done. So, the authors decided use the SBM model for efficiency calculation in this research .

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Table 1 DEA models

Model name (Creator)	**Orientation**	**Returns to scale**	**Key feature / purpose**
CCR (Charnes, Cooper, Rhodes, 1978)	Input or Output	Constant (CRS)	The foundational radial model. Measures overall technical efficiency.
BCC (Banker, Charnes, Cooper, 1984)	Input or Output	Variable (VRS)	Measures pure technical efficiency by isolating scale efficiency.
Additive (Charnes et al., 1985)	Non-oriented	CRS or VRS	Maximizes the sum of input and output slacks. Focuses directly on slack improvement.
SBM (Slacks-Based Measure) (Tone, 2001)	Non-oriented (Primarily)	CRS or VRS	Non-radial, slack-based model providing a standardized efficiency score between 0 and 1.
Super-SBM (Tone, 2002)	Non-oriented	CRS or VRS	An extension of SBM for ranking efficient DMUs (efficiency score can be >1).
Window-DEA (Klopp, 1985)	Input/Output/Non	CRS or VRS	Analyzes efficiency over time by treating a DMU in different periods as different entities.
Network DEA (Färe & Grosskopf, 2000)	Input/Output/Non	CRS or VRS	Models the internal structure of a DMU (e.g., processes, stages).
RAM (Range Adjusted Measure) (Cooper et al., 1999)	Non-oriented	CRS or VRS	A slack-based model that normalizes slacks by their data range.

2 Literature review

Facility location for emergency services focuses on minimizing response times and maximizing coverage, integrating transportation networks and stochastic elements. Recent models (2020–2025) emphasize temporary facilities for humanitarian logistics, addressing dynamic demands in disasters [1, 2]. Set-Covering Problem is NP-hard problem aims to cover demand with minimal facilities, with applications in emergencies through hybrid variants like location-routing.

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Advancements include metaheuristics for large instances [3, 4]. DEA evaluates efficiency of units like emergency departments, using inputs like staff and outputs like patient throughput. Hybrid models combine DEA with coverage for optimal siting [5, 6]. Fuzzy approaches handle vagueness in data, merging set-covering and DEA for robust models in ambiguous environments, enhancing resilience in emergency planning [7, 8]. The location of emergency and rescue facilities is a critical area in operations research and optimization, aiming to optimize placement to minimize response times, maximize coverage, and adapt to uncertainties in demand and infrastructure. Recent literature (2021–2025) highlights the shift toward temporary and modular facilities in humanitarian logistics, particularly for sudden-onset disasters like earthquakes [1]. Systematic reviews classify problems into discrete and continuous models, with discrete models prevalent due to computational efficiency [9]. For routine emergencies managed by emergency medical services (EMS), fire, and police, coverage-based models integrate transportation networks via distance, routing, accessibility, and stochastic travel times [10]. Models such as location-routing combine siting with vehicle paths, while equilibrium-constrained programs address network interactions under degradation [11]. Challenges include data-driven optimization and machine learning integration for predictive analytics [12]. Applications encompass urban EMS optimization considering population density and budgets, with hybrid evaluations proposing new sites to enhance coverage [13]. Multi-objective models target stations and transfers, incorporating multi-period planning with backups [14].

Future directions advocate maximal survival models, dynamic redeployment, and robust optimization against disruptions, leveraging big data for multi-level networks in large-scale emergencies [15].

Table 2 Models comparisons

Model type	Key features	Applications	Limitations
Location-Routing	Integrates siting and routing; stochastic times	Urban EMS deployment	Computational complexity
Coverage-Based (e.g., MCLP)	Minimizes distances; discrete	Fire stations	Fixed demand assumptions
Modular/Temporary	Relocation, capacity adjustment	Humanitarian shelters	Understudied closure decisions
Robust/Stochastic	Handles uncertainty in demand/time	Post-disaster medical centers	Data quality sensitivity

2.1 Set-covering problem

SCP is a fundamental NP-hard combinatorial optimization problem in operations research, aiming to select the minimum number or cost of subsets to cover all elements in a universe [3]. Historical developments date back to the 1960s–1970s, evolving with heuristics for large instances [16]. Variants include total/partial covering, probabilistic, and set-covering routing problems (SCRPs), integrating covering with routing for logistics and disaster applications [4].

In emergency contexts, SCP optimizes facility siting for coverage within thresholds, with extensions like anti-covering for undesirable sites. Solution methods range from exact (Lagrangian relaxation) to meta-heuristics (genetic algorithms) [17]. Recent developments (2024–2025) include classification schemes for SCRPs and applications in healthcare and relief.

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Table 3 Covering location types

Variant	Objective	Constraints	Solution Approaches
Total Covering (TCP)	Minimize facilities for full coverage	Each demand covered at least once	Dynamic programming, branch-and-bound
Partial Covering (MCP)	Maximize covered demands with fixed facilities	Limited number of sites	Greedy heuristics, LP relaxation
Capacitated SCP	Minimize cost with capacity limits	Workload and backup requirements	Meta-heuristics like genetic algorithms
Probabilistic SCP	Handle uncertain coverage	Probability thresholds	Stochastic programming, fuzzy extensions

2.2 DEA in emergency facility location

DEA is a non-parametric method for evaluating relative efficiency of DMUs using multiple inputs and outputs [18]. Narrative reviews cluster applications into basic models (variable/constant returns to scale), advanced hybrids (super-efficiency, slack-based), time-series (window analysis, Malmquist index), and integrations with simulations or machine learning [19].

In emergency location, DEA hybrids with coverage models optimize sites, as in assessing primary health centers or stroke treatments [20]. Basic DEA models evaluate the efficiency of emergency departments using inputs like staff and beds, outputs like patient visits [21]. Challenges include low discrimination power, addressed by multi-criteria variants [22]. Trends (2023–2025) integrate AI for pandemics and dynamic efficiency [23].

Table 4 DEA models inputs & outputs

DEA model type	Inputs examples	Outputs examples	Emergency applications
Basic (VRS/CRS)	Staff, beds, budget	Patient throughput, discharges	Efficiency scoring in EDs for resource optimization
Advanced (Super-Efficiency, SBM)	Crashes, population density	Coverage share, treatment rates	Hybrid with MCLP for new facility siting
Time-Series (MPI, Window)	Time-varying resources	Longitudinal performance	Benchmarking stroke/AMI treatments over time
Integrated (with ML/Simulation)	Wait times, transfers	Reduced futile transfers	Overcrowding mitigation in high-demand EDs

2.3 Set-covering and DEA in fuzzy environments

Combining SCP with DEA in fuzzy environments addresses imprecision in data, such as uncertain demands or efficiencies [24]. Fuzzy SCP extends classical models by defining fuzzy covers through membership functions, reducing to nonlinear integer programs [25]. Fuzzy DEA evaluates efficiencies with fuzzy inputs/outputs, using possibility measures for ranking [26].
Integrated models like multi-criteria DEA (MCDEA) incorporate efficiency into fuzzy settings for location-allocation [27]. In uncertain environments, models use uncertainty theory for response times, with maximal covering variants under confidence levels [29]. Case studies in urban emergencies employ fuzzy multi-objective functions with NSGA-II for Pareto fronts [29].

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Table 5 Combination approaches

Combined approach	Fuzzy elements	Key models	Benefits in emergency location
Fuzzy SCP	Fuzzy covers, membership functions	Nonlinear IP reduction	Handles uncertain coverage thresholds
Fuzzy DEA	Fuzzy inputs/outputs, possibility measures	Aggregation for ranking	Efficiency evaluation under data vagueness
MCDEA with Allocation	Fuzzy parameters, multi-objectives	FPP and deviation method	Flexible facility numbers, improved assignment
Uncertain Covering	Uncertainty distributions, confidence levels	(α, β)-maximal models	Robust against indeterminate times/demands

3 Methodology

Given the critical importance of facility location (particularly in healthcare and emergency services, where criteria such as cost, customer satisfaction, and operational efficiency must be optimized) and considering the inherent uncertainty of real-world data, this study was undertaken. The primary objective is to develop a novel mathematical model that integrates SCP and DEA within a fuzzy environment for relief and emergency facility location. Mathematical modeling provides an effective tool for determining the placement of emergency facilities. On one hand, the selected sites must guarantee full coverage of the study area while meeting the efficiency standards defined by the relevant authority. On the other hand, because a substantial portion of available data in healthcare and emergency contexts is inherently fuzzy, a key research question emerges: Can SCP (used to identify candidate sites ensuring complete coverage) and DEA (used to evaluate efficiency) be jointly applied and solved under a fuzzy framework?

A review of the literature indicates that no prior study has addressed emergency facility location by combining SCP and DEA in a fuzzy environment. This gap highlights the novelty of the present research, particularly in light of the growing reliance on fuzzy logic for modeling real-world decision-making, where both coverage and efficiency must be simultaneously ensured.

Among the four models identified in the literature review, the first (SCP) is employed as the foundation of this study, as it determines the number of sites required to achieve full coverage. Building upon this, an efficiency assessment using DEA models is incorporated, relying on uncertain real-world data obtained from the Iranian Red Crescent Society in Fars Province, which serves as the responsibility organization in our case study. The proposed model is then implemented in GAMS software, producing a set of optimal sites that cover the designated area as effectively as possible under the given constraints.

The research methodology involves constructing a decision matrix for each region based on the indicators listed in Table 6. These indicators were derived from the expertise and professional judgment of domain specialists.

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Table 6 Experts' index

NO.	Index	Importance coefficient
1	Access to consumption resources (water, electricity, gas)	0.082
2	Security and safety	0.081
3	Radio Communication ease	0.080
4	Distance to the nearest flood	0.079
5	Quick and easy access to accident-prone areas and routes	0.077
6	The distance to the nearest emergency station (EMS)	0.073
7	Distance to the Nearest Police Station and Highway Maintenance Depot	0.072
8	Distance to the nearest gas station	0.071
9	Absence of Post and Unconventional Elevations	0.070
10	Available Square Footage and Ownership	0.069
11	Distance to the Nearest Medical Center	0.068
12	Development Potential	0.067
13	Population Density of the Region	0.058
14	Distance to Industrial zones such as Factories	0.053
TOTAL	1	

After constructing the decision matrix, the data are normalized into quantitative and dimensionless values through the linear scaling approach. It should be emphasized that certain indicators exhibit a negative orientation and, therefore, conflict with positively oriented ones. For example, Index 4 is inversely related to Index 11. To address this inconsistency, the values of negatively oriented indicators are first transformed (inverted) and subsequently normalized.

In the next stage, Multiple Attribute Decision-Making (MADM) techniques are applied. In this study, the TOPSIS method is utilized to select a minimum of three representative points within each region. These points are then considered as candidate locations within the SCP framework and are further evaluated at the set level. Finally, the efficiency of the identified candidate points is measured using SBM.

The Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) is one of the most widely applied methods in MADM. It is based on the principle that the best alternative should have the shortest distance from the positive ideal solution (PIS) and the farthest distance from the negative ideal solution (NIS).

In practice, TOPSIS involves:

1. Constructing a normalized decision matrix.
2. Weighting the criteria.
3. Identifying the PIS and NIS.
4. Calculating the Euclidean distance of each alternative from both PIS and NIS.
5. Ranking alternatives based on their relative closeness to the ideal solution.

This makes TOPSIS particularly effective for decision problems where trade-offs among multiple conflicting criteria are required.

In summary, TOPSIS is a powerful MADM tool that evaluates alternatives based on their relative closeness to the ideal solution. Its balance of simplicity, interpretability, and efficiency explains its popularity in fields like facility location, supply chain, and disaster management.

Model (1) -presented in the previous section- must be linearized to be used in the proposed model. After linearization, it can be seen as SBM model is structured as follows:

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$$
\begin{aligned}
& Min \;\; z = q - \frac{1}{m}\sum_{i=1}^{m}\frac{s_i^-}{R_{ip}^-} \qquad\qquad (2)\\
& s.t. \qquad q + \frac{1}{s}\sum_{r=1}^{s}\frac{s_r^+}{R_{rp}^+} = 1\\
& \qquad\qquad \sum_{j\in J}\lambda_j x_{ij} + s_i^- = q x_{ip} \;\;,\; i = 1,\dots,m\\
& \qquad\qquad \sum_{j\in J}\lambda_j y_{rj} - s_r^+ = q y_{rp} \;\;,\;\; r = 1,\dots,s\\
& \qquad\qquad \sum_{j\in J}\lambda_j = q\\
& \qquad\qquad \lambda \ge 0 \;,\; S^- \ge 0 \;,\; S^+ \ge 0,\; q \ge \varepsilon.
\end{aligned}
$$

In this model, the following items are model's inputs:

1. Light and heavy rescue hydraulic equipment.
2. The number of station rescuers and employees.
3. The number of training courses in each station.

And, the following items are model's outputs:

1. The medical services that provided to clients.
2. The number of aided incidents.
3. The number of employee certificates that presented by relief organization.

The above model has been used in the proposed model of this research. Finally, our proposed model is as follows.

Variables:

E_j : A binary decision variable taking the value 1 if candidate site j is selected, and 0 otherwise.

$\overline{N}, \acute{N}$: Deflective variables

Sets and Indices:

J : candidate points {1,…,n}

J_t :Subset of candidate sites located within county *t*

$j_1 \cup j_2 \cup j_3 \cup \dots \cup j_T = J$ {1,…, T}

r: Set of outputs for DMUs {1,…, s}

i :Set of inputs for DMUs {1,…, m}

p :Index of the DMU under evaluation

k :Index of the indicator {1,…,14}

Parameters:

W_{kj} :Score of the k-th indicator for candidate point *j*

W_j : Score of candidates point *j*

A_j : Demand related to candidate point *j*

C_j : Cost of establishing candidate point *j*

$d_{jj'}$:Spatial distance between two candidate points *j* and *j′*

tv_j : Traffic volume (potential danger) of the area of candidate point *j*

Pd_j : Population density of the area related to candidate point *j*

$\overline{t_j}$:Average time to reach the incident site from point j (shortest time)

a'_j :Accessibility to incident-prone areas from point j
α_p : Expected efficiency value related to the candidate point under evaluation
x_{ij} : Input i for $DMUj$
y_{rj} : Output r for $DMUj$

Scalars:

B: Total available budget for establishing rescue and relief stations.
H: Maximum number of rescue and relief stations (both established and don't established)
L: Number of existing established stations.
W: Minimum acceptable score for candidate point selection.
D: Maximum coverage distance for candidate points (coverage radius)
Ac: Minimum required demand for candidate point selection
TV: Minimum potential danger
POD: Minimum population density for candidate point selection
MIT: Maximum allowable response time to reach an incident site.
ACS: Standard level of accessibility to incident-prone areas (5-10)
β: Minimum efficiency for candidate points selection

$$Max\ z_4 = z_1 - z_2 + z_3 \tag{3}$$

$$s.t.\quad z_1 = \sum_{j\in J} W_j E_j \tag{4}$$

$$z_2 = \bar{N} - N' \tag{5}$$

$$z_3 = \sum_{j\in J} \alpha_j E_j \tag{6}$$

$$W_{kj} E_j \geq W'_k E_j \qquad j \in J, k = 1,...,14 \tag{7}$$

$$\sum_{J\in j} E_j \leq H - L \tag{8}$$

$$\frac{d_{jj'}}{2} \geq D.E_j E_{j'} \qquad j', j \in J \quad , j \neq j' \tag{9}$$

$$\sum_{J\in j} C_j E_j + \bar{N} - N' = \beta \tag{10}$$

$$\tilde{A}_j E_j \geq Ac.E_j, \qquad j \in J \tag{11}$$

$$\sum_{j\in J_t} E_j \leq 1 - L_t \qquad t = 1,...,T \tag{12}$$

$$tv.E_j \geq TV.E_j, \qquad j \in J \tag{13}$$

$$pd_j.E_j \geq POD.E_j, \qquad j \in J \tag{14}$$

$$\tilde{\tilde{t}}_J.E_j \leq MIT.E_j, \qquad j \in J \tag{15}$$

$$a'.E_j \leq Acs.E_j, j \in J \tag{16}$$

$$\alpha_p = q_p - \frac{1}{m}\sum_{i=1}^{m}\frac{\bar{S}_{ip}}{\bar{R}_{ip}} \qquad p \in J \tag{17}$$

$$q_p + \frac{1}{S}\sum_{r=1}^{s}\frac{\overset{+}{S}_{rp}}{\overset{+}{R}_{ip}} = 1 \qquad p \in J \tag{18}$$

$$\sum_{j \in J} E_j \lambda_j x_{ij} + \bar{S}_{ip} = q_p x_{ip} \qquad p \in J,\ \mathrm{i} \in \mathrm{I} \tag{19}$$

$$\sum_{j \in J} E_j \lambda_j y_{rj} - \overset{+}{S}_{rp} = q_p y_{rp} \qquad p \in J,\ \mathrm{r} \in \mathrm{R} \tag{20}$$

$$\sum_{j \in J} E_j \lambda_j = q_p \qquad p \in J \tag{21}$$

$$E_j \in \{0,1\}, \qquad j \in J \tag{22}$$

$$\bar{N} \geq 0, \quad N' \geq 0 \tag{23}$$

$$q \geq \varepsilon, \ \ S^- \geq 0, \quad S^+ \geq 0, \quad \lambda \geq 0 \tag{24}$$

Objective Function:
(3): The objective function, essentially the sum of the first three Constraints.
Constraints:
(4): This constraint of the model, expressing the maximization of the score for selected candidates.
(5): This constraint, expressing the minimization of deviation variables.
(6): This constraint, expressing the maximization of the efficiency of selected points.
(7): This constraint, ensuring that the selected point has a minimum acceptable score.
(8): This constraint, ensuring that the selected points do not include existing points.
(9): This constraint guarantees the coverage radius of the selected points. In fact, this constraint ensures that two points are not selected within the coverage radius.
(10): This constraint shows that the total cost of establishing selected points is equal to the allocated budget for establishing selected points.
(11): This constraint shows that the selected points must have the minimum required demand.
(12): This constraint shows that in each county, the establishment of at least one rescue and relief station is required. If no active station exists within a county, at least one of the proposed candidate locations should be selected.
(13): This constraint shows that the selected point must have a minimum potential danger.
(14): This constraint shows that the selected point must have a minimum population density within its coverage radius.
(15): This constraint is related to the minimum time to reach the incident site. So, the selected point must have a maximum time to reach the incident site; otherwise, it won't be selected.
(16): This constraint ensures that the level of access to incident-prone areas for the selected point is less than the standard level of access.
(17), (18), (19), (20) and (21): these constraints according to the SMB Model endure for candidate points efficiency calculation.
(22), (23) and (24): These constraints indicate the bounds of the model variables.

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In order to solve the problem, using the following relations, fuzzy constraints are converted into non-fuzzy constraints:

$$\tilde{a} = (l, m, u) \ , \ \tilde{b} = (l', m', u') \ , \ \tilde{a} \le \tilde{b} \quad \Rightarrow \quad \begin{cases} l \le l' \\ m \le m' \\ u \le u' \end{cases}$$

So:

$$\sum_{j=1}^{n} \tilde{a}_{ij} x_j \le \tilde{b}_i \quad \Rightarrow \quad \begin{cases} \sum_{j=1}^{n} l_{a_{ij}} x_j \le l_i \\ \sum_{j=1}^{n} m_{a_{ij}} x_j \le m_i \\ \sum_{j=1}^{n} u_{a_{ij}} x_j \le u_i \end{cases}$$

So, the constraint (11) converted to following equations:

$l_{A_j} E_j \ge Ac.E_j$

$m_{A_j} E_j \ge Ac.E_j$

$u_{A_j} E_j \ge Ac.E_j$

The constraints (13), (14), (15) and (16) converted too.

4 Case study and practical example

This section presents the findings of the study. The results are reported alongside charts, and statistical analyses, accompanied by detailed interpretation and discussion.

Fars Province in Iran is recognized as one of the most accident-prone regions in the country in terms of road traffic incidents. Given its geographical vastness and the large number of counties, the establishment and spatial distribution of road rescue stations hold particular importance. The northern counties of Fars Province, due to their proximity to central provinces, their connection to major industrial zones, and their location along critical north–south transit corridors, exhibit heightened significance in this regard.

Data obtained from the Red Crescent Organization of Fars Province confirm that these areas consistently record high accident rates, thereby underscoring the urgent need for effective rescue and relief infrastructure. Establishing an adequate and optimally distributed number of road rescue stations is therefore essential to improving the quality and timeliness of services provided in response to road accidents.

In accordance with the adopted research methodology, expert-proposed candidate locations for new rescue stations were first identified. Subsequently, these proposed points were systematically evaluated and analyzed through the selected optimization model, following the process outlined below.

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4.1 Evaluation of proposed points

In each county located in the northern part of Fars Province, Iran, five candidate points were proposed by domain experts. These proposed points (five per county) were evaluated based on the criteria listed in Table 6. The results of this evaluation are summarized in Table 7, which presents the assessment of each candidate point for a given county. A separate table was prepared for each county in northern Fars Province.

Table 7 Proposed points information (decision matrix)

Importance coefficient	0.082	0.081	0.08	0.079	0.077	0.073	0.072	0.071	0.07	0.069	0.068	0.067	0.058	0.053
Index	**Access to Consumable Resources (Water, Electricity, Gas)**	**Security and safety**	**Radio Communication Ease**	**Distance to the Nearest flood (in kilometers)**	**Quick and Easy Access to accident-Prone Areas and Routes**	**Distance to the Nearest Emergency Station (EMS)(Kilometers)**	**Distance to the Nearest Police Station and Highway Maintenance Depot**	**Distance to the Nearest Gas Station**	**Absence of Unconventional elevation**	**Available Square Footage and Ownership**	**Distance to the nearest medical center**	**Development potential**	**Population Density of the Region**	**Distance to industrial zones such as factories**
Point 1	Very high	High	Very high	20	Very high	5	5	5	Medium	1000	25	Relativel very y low	1000	45
Point 2	Medium	Relatively high	Relatively very low	25	Mediu m	7	12	8	Very low	1250	35	Very high	1500	55
Point 3	Low	high Very	Medium	20	Relati vely very high	12	5	12	Very high	1450	15	Medium	1700	45
Point 4	Very high	high Very	Relatively very high	15	Relati vely high	8	4	10	high	750	35	Relativel low y	1600	20
Point 5	Relatively high	Relatively very low	Relatively very high	30	Very high	2	8	9	Medium	800	10	Medium	1200	15

4.2 Selection of optimal points using the TOPSIS method

For each county, a decision matrix was constructed based on the criteria in Table 7. The data were then normalized to become dimensionless. It should be noted that some indicators have a negative nature, conflicting with other positive indicators. To address this issue, the values corresponding to negative indicators were first inverted before normalization. Furthermore, due to heterogeneity among the indicators, a homogenization process was applied. The results of the TOPSIS analysis are presented in Table 8, prepared separately for each county.

Table 8 Homogenized decision matrix

Index	Access to Consumable Resources (Water, Electricity, Gas)	Security and safety	Radio Communication Ease	Distance to the Nearest flood (in kilometers)	Quick and Easy Access to accident-Prone Areas and Routes	Distance to the Nearest Emergency station (EMS)(Kilometers)	Distance to the Nearest Police Station and Highway Maintenance Depot	Distance to the Nearest Gas Station	Absence of Unconventional elevation	Available Square Footage and Ownership	Distance to the nearest medical center	Development potential	Population Density of the Region	Distance to industrial zones such as factories
Point 1	0.049	0.025	0.013	0.04	0.033	0.027	0.018	0.021	0.027	0.037	0.019	0.039	0.026	0.031
Point 2	0.038	0.038	0.059	0.03	0.02	0.005	0.028	0.051	0.021	0.02	0.014	0.009	0.022	0.017
Point 3	0.043	0.043	0.033	0.02	0.048	0.049	0.042	0.021	0.043	0.033	0.023	0.022	0.033	0.01
Point 4	0.016	0.041	0.026	0.05	0.025	0.038	0.035	0.017	0.043	0.024	0.042	0.035	0.033	0.031
Point 5	0.027	0.031	0.033	0.03	0.04	0.027	0.032	0.034	0.011	0.037	0.042	0.035	0.007	0.021

Owing to the large amount of information and the number of tables, only the results for one county are presented here. Based on the TOPSIS rankings, the following points were identified as the top three candidates in each county:

County 1: Points 1, 2 and 3
County 2: Points 2, 1 and 3
County 3: Points 1, 5 and 2
County 4: Points 4, 2 and 1
County 5: Points 3, 1 and 2
County 6: Points 4, 3 and 1
County 7: Points 1 and 2

In total, 20 candidate points qualified to enter the proposed model.

The proposed model was then solved using these 20 candidate points in GAMS software. The results demonstrate that the optimal selection of points for establishing roadside emergency and rescue stations was achieved in such a way that both efficiency and coverage across all counties are ensured.

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Table 9 result of solving proposed model

Variables	E_1	E_2	E_3	E_4	E_5	E_6	E_7	E_8	E_9	E_{10}	E_{11}	E_{12}	E_{13}	E_{14}	E_{15}	E_{16}	E_{17}	E_{18}	E_{19}	E_{20}	Z^*
Result	0	0	0	0	1	0	0	0	0	1	0	0	0	1	0	0	0	0	1	0	-264

As can be observed from the above results in Table 9, points 5, 10, 14 and 19 are selected for the establishment of rescue and relief stations, providing coverage to the entire northern region of Fars province in Iran.

5 Discussion and conclusion

The results obtained from solving the proposed SCP model indicate that the framework developed for locating road rescue stations in a fuzzy environment demonstrates superior efficiency compared to existing approaches. The analysis reveals that the selected sites optimize the objective function in the most effective manner, while simultaneously ensuring comprehensive coverage across the entire north of the province for rescue and relief service delivery. The allocation of stations derived from the proposed model adequately meets regional demands and aligns with the operational guidelines of relief organizations.

Given the inherent uncertainty in real-world data, the fuzzy-based approach enhances the realism and practical applicability of the results. Comparative analysis with previous studies further confirms the efficiency and optimality of the model in identifying suitable locations for rescue facilities.

For future research, it is recommended to investigate the integration of alternative DEA models for efficiency evaluation. In addition, MCLP may be considered in special cases where maximizing service coverage is of primary concern.

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