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Optimal Two Stages Specially Structured Flow Shop Scheduling: Minimize the Rental Cost with Independent Setup Time

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Abstract One of the earliest results in flow shop scheduling theory is an algorithm given by Johnson's [1] for scheduling jobs on two or three machines to minimize the total elapsed time whenever the processing times of jobs are random. The present paper is an attempt to develop a heuristic algorithm for two stages specially structured flow shop scheduling in which the processing times of the jobs are not completely random, but bear a well defined relationship to one another to minimize the utilization time of machines and hence their rental cost under a specified rental policy. Further the processing times and independent set up times, each are associated with probabilities. A computer programme followed by a numerical illustration is given to validate the proposed algorithm.

Keywords Processing Time, Set Up Time, Specially Structured Flow Shop, Makespan, Utilization Time, Rental Cost.

1 Introduction

Flow shop scheduling problem concerns with the sequencing of jobs through a series of machines in exactly the same order with aim to optimize a number of objectives such that some performance criterion is maximized or minimized. In modern manufacturing and operations management, the minimization of utilization time/rental cost of machines is the significant factors as for the reason of upward stress of competition on the markets. In most of literature the processing time of the machines are considered to be random. There are cases when the processing time of jobs are not random but follow some well defined structural conditions. In such case we can have different heuristic approach to find the algorithm(s) alternative and proficient as compared to the existing algorithm(s) to minimize the utilization time of the machines and hence their rental cost. The majority of scheduling research assumes setup as negligible or part of processing time. While this assumption adversely affects the solution quality for many applications which require explicit treatment of set up. Such applications, coupled with the emergence of product concept like time based competitions and

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group technology, have motivated to include setup considerations in scheduling theory. A flow shop scheduling problem has been one of the classical problems in production scheduling since Johnson [1] proposed the well known Johnson's rule in the two stage flow shop makespan scheduling problem. Gupta, J.N.D [2] gave an algorithm to find the optimal schedule for specially structured flowshop scheduling. Yoshida & Hitomi [3] studied optimal two stage production schedule with separated setup time. Bellman [4] discussed the mathematical as pects of scheduling theory. Ignall & Schrage [5] introduces branch and bound technique to flow shop scheduling problems. Bagga [6] studied sequencing in rental situations. Szware [7] discussed some special cases of the flow shop problems. Singh, T.P. [8] introduced the concepts of job block, Transportation time and break down machine times in two stage flow shop scheduling problems. Gupta [9] studied two stage hybrid flow shop scheduling. Narain and Bagga [10] introduced two machine flow shop problem with availability constraint on each machine. Singh & Gupta [11, 12] discussed the minimization of rental cost in two stage flow shop scheduling when the processing times are associated with probabilities. Gupta & Sharma [13] introduced the concept of breakdown of the machines in minimization of rental cost of machines in two stage flowshop.

Gupta, Sharma & Shashi [14] studied specially structured two stage flow shop scheduling model to minimize the rental cost in which the processing times are associated with their corresponding probabilities. The present work is an attempt to extend their study by introducing independent set up time with their corresponding probabilities to minimize the utilization time of the machines and hence their rental cost under specified rental policy. The proposed algorithm is more efficient and less time consuming as compared to Johnson's [1] algorithm to minimize the utilization time of machines and hence their rental cost for specially structured flow shop scheduling.

2 Practical situation

Many applied and experimental situations exist in our day-to-day working in factories and industrial production concerns when one has got the assignments but does not have one's own machine or does not have enough money or does not want to take risk of investing huge amount of money to purchase machine. Under such circumstances, the machine has to be taken on rent in order to complete the assignments. For example, In his starting career, we find a medical practitioner does not buy expensive machines say X-ray machine, the Ultra Sound Machine, Rotating Triple Head Single Positron Emission Computed Tomography Scanner, Patient Monitoring Equipment, and Laboratory Equipment etc., but instead takes on rent. Rental of medical equipment is an affordable and quick solution for hospitals, nursing homes, physicians, which are presently constrained by the availability of limited funds due to the recent global economic recession. Renting enables saving working capital, gives option for having the equipment, and allows upgradation to new technology. Setup time includes work to prepare the machine, process or bench for product parts or the cycle. This includes obtaining tools, positioning work-in-process material, return tooling, cleaning up, setting the required jigs and fixtures, adjusting tools and inspecting material and hence significant.

3 Notations

The following notations are used through the course of present paper:

S	:	Sequence of jobs 1, 2, 3, …, n
a_{ij}	:	Processing time of i^{th} job on machine M_j
p_{ij}	:	Probability associated to the processing time a_{ij}
s_{ij}	:	Set up time of i^{th} job on machine M_j
q_{ij}	:	Probability associated to the set up time s_{ij}
A_{ij}	:	Expected processing time of i^{th} job on machine M_j
S_{ij}	:	Expected set up time of i^{th} job on machine M_j
$A'_{i,j}$	:	Expected flow time of i^{th} job on machine M_j
$t_{ij}(S_k)$	:	Completion time of i^{th} job of sequence S_k on machine M_j
$I_{ij}(S_k)$	:	Idle time of machine M_j for job i in the sequence S_k
$U_j(S_k)$	:	Utilization time for which machine M_j
$R(S_k)$	:	Total rental cost for the sequence S_k of all machine
$CT(S_k)$	:	Total completion time of jobs for the sequence S_k
C_j	:	Rental cost of machine M_j.

4 Rental Policy (P)

The machines will be taken on rent as and when they are required and are returned as and when they are no longer required i.e. the first machine will be taken on rent in the starting of the processing the jobs and 2nd machine will be taken on rent at time when 1st job is completed on 1st machine.

Definition 1. Completion time of i^{th} job on machine M_j is denoted by t_{ij} and is defined as $t_{ij} = max\ (t_{i-1,j} + s_{(i-1),j} \times q_{(i-1),j}\ ,\ t_{i,j-1}) + a_{ij} \times p_{ij}$ for $j \geq 2$.

$= max\ (t_{i-1,j} + S_{i-1,j}\ ,\ t_{i,j-1}) + A_{i,j}$

where $A_{i,j}$= expected processing time of i^{th} job on machine j

$S_{i,j}$= expected set up time of i^{th} job on machine j.

5 Problem Formulation

Let some job i (i = 1, 2, …, n) be to be processed on two machines M_j (j = 1,2) under the specified rental policy P. Let a_{ij} be the processing time of i^{th} job on j^{th} machine with probabilities p_{ij} and s_{ij} be the setup time of i^{th} job on j^{th} machine with probabilities q_{ij}. Let A_{ij} be the expected processing time and $S_{i,j}$ be the expected setup time of i^{th} job on j^{th} machine. Our aim is to find the sequence $\{S_k\}$ of the jobs which minimize the rental cost of all the machines in this two stage specially structured flowshop scheduling problem.

The mathematical model of the problem in matrix form is as shown in table 1.

Table 1 The mathematical model of the problem in matrix form

Jobs	Machine A				Machine B			
i	a_{i1}	p_{i1}	s_{i1}	q_{i1}	a_{i2}	p_{i2}	s_{i2}	q_{i2}
1	a_{11}	p_{11}	s_{11}	q_{11}	a_{12}	p_{12}	s_{12}	q_{12}
2	a_{21}	p_{21}	s_{21}	q_{21}	a_{22}	p_{22}	s_{22}	q_{22}
3	a_{31}	p_{31}	s_{31}	q_{31}	a_{32}	p_{32}	s_{32}	q_{32}
4	a_{41}	p_{41}	s_{41}	q_{41}	a_{42}	p_{42}	s_{42}	q_{42}
-	-	-	-	-	-	-	-	-
-	-	-	-	-	-	-	-	-
n	a_{n1}	p_{n1}	s_{n1}	q_{n1}	a_{n2}	p_{n2}	s_{n2}	q_{n2}

Mathematically, the problem is stated as:
Minimize $R(S_k) = t_{n,1}(S_k)\times C_1 + U_2(S_k)\times C_2$
Subject to constraint: Rental Policy (P)
Our objective is to minimize rental cost of machines while minimizing the utilization time.

6 Theorems

The following theorems have been proved to get the optimal sequence of jobs processing.

Theorem 1. If $A'_{i1} \le A'_{j2}$ *for all i, j, i ≠ j,* then $k_1, k_2 \ldots\ldots k_n$ is a monotonically decreasing sequence, where $k_n = \sum_{i=1}^{n} A'_{i1} - \sum_{i=1}^{n-1} A'_{i2}$; $A'_{i1} = A_{i1} - S_{i2}$ and $A'_{j2} = A_{j2} - S_{j1}$.

Solution: Let $A'_{i1} \le A'_{j2}$ for all i, j , $i \ne j$ i.e., max $A'_{i1} \le$ min A'_{j2} for all i, j ; $i \ne j$

Let $k_n = \sum_{i=1}^{n} A'_{i1} - \sum_{i=1}^{n-1} A'_{i2}$

Therefore, we have $k_1 = A'_{11}$

Also $k_2 = A'_{11} + A'_{21} - A'_{12} = A'_{11} + (A'_{21} - A'_{12}) \le A'_{11}$ $(\because A'_{21} \le A'_{12})$

$\therefore k_1 \le k_2$

Now, $k_3 = A'_{11} + A'_{21} + A'_{31} - A'_{12} - A'_{22}$

$= A'_{11} + A'_{21} - A'_{12} + (A'_{31} - A'_{22}) = k_2 + (A'_{31} - A'_{22}) \le k_2$ $(\because A'_{31} \le A'_{22})$

Therefore, $k_3 \le k_2 \le k_1$ or $k_1 \ge k_2 \ge k_3$.

Continuing in this way, we can have $k_1 \ge k_2 \ge k_3 \ge ,\ldots, \ge k_n$, a monotonically decreasing sequence.

Corollary 1. The total rental cost of machines is same for all the possible sequences.
Proof. The total elapsed time

$$T(S) = \sum_{i=1}^{n} A_{i2} + \sum_{i=1}^{n-1} S_{i2} + A_{11} = \text{Constant for all sequences.}$$

It implies that under rental policy P, the total elapsed time remains constant. Therefore total rental cost of machines is same for all the sequences.

Theorem 2. If $A'_{i1} \ge A'_{j2}$ for all *i, j, i ≠ j*, then $k_1, k_2, \ldots, k_n$ is a monotonically increasing sequence, where. $k_n = \sum_{i=1}^{n} A'_{i1} - \sum_{i=1}^{n-1} A'_{i2}$; $A'_{i1} = A_{i1} - S_{i2}$ and $A'_{j2} = A_{j2} - S_{j1}$.

Proof. Let $k_n = \sum_{i=1}^{n} A_{i1}^{'} - \sum_{i=1}^{n-1} A_{i2}^{'}$

Let $A^{'}_{i1} \geq A^{'}_{j2}$ for all i, j , $i \neq j$ i.e., min $A^{'}_{i1} \geq$ max $A^{'}_{j2}$ for all i, j , $i \neq j$

Here $k_1 = A^{'}_{11}$

$k_2 = A^{'}_{11} + A^{'}_{21} - A^{'}_{12} = A^{'}_{11} + (A^{'}_{21} - A^{'}_{12}) \geq k_1$ ($\because$ $A^{'}_{21} \geq A^{'}_{j2}$)

Therefore, $k_2 \geq k_1$.

Also, $k_3 = A^{'}_{11} + A^{'}_{21} + A^{'}_{31} - A^{'}_{12} - A^{'}_{22} = A^{'}_{11} + A^{'}_{21} - A^{'}_{12} + (A^{'}_{31} - A^{'}_{22})$

$= k_2 + (A^{'}_{31} - A^{'}_{22}) \geq k_2$ ($\because A^{'}_{31} \geq A^{'}_{22}$)

Hence, $k_3 \geq k_2 \geq k_1$.

Continuing in this way, we can have $k_1 \leq k_2 \leq k_3, ..., \leq k_n$, a monotonically increasing sequence.

Corollary 2. The total rental cost of machines is same for all the possible sequences.

Proof. The total elapsed time $=T(S)$

$$= \sum_{i=1}^{n} A_{i2} + \sum_{i=1}^{n} S_{i2} + \max_{1 \leq i \leq n} \{k_i\}$$

$$= \sum_{i=1}^{n} A_{i2} + \sum_{i=1}^{n} S_{i2} + k_n = \sum_{i=1}^{n} A_{i2} + \sum_{i=1}^{n} S_{i2} + \left(\sum_{i=1}^{n} A_{i1}^{'} - \sum_{i=1}^{n-1} A_{i2}^{'} \right)$$

$$= \sum_{i=1}^{n} A_{i2} + \sum_{i=1}^{n} S_{i2} + \left(\sum_{i=1}^{n} A_{i1} - \sum_{i=1}^{n} S_{i2} - \sum_{i=1}^{n-1} A_{i2} + \sum_{i=1}^{n-1} S_{i1} \right)$$

$$= \sum_{i=1}^{n} A_{i1} + \sum_{i=1}^{n-1} S_{i1} + A_{n2} = \text{Constant for all sequences.}$$

It implies that under rental policy P the total elapsed time is constant for all sequences for machine M_2. Therefore total rental cost of machines is same for all the sequences.

7 Algorithm

The following algorithm is developed to find the optimal sequence of jobs processing minimizing the total rental cost of machines under specified rental policy for two stage specially structured flowshop scheduling problem.

Step 1: Calculate the expected processing times, $A_{ij} = a_{ij} \times p_{ij} \forall i, j.$ and expected setup time $S_{ij} = s_{ij} \times q_{ij} \forall i, j.$

Step 2: Define the two fictitious machines G and H with processing time $A_{i1}^{'}$ and $A_{i2}^{'}$ defined as follows: $A_{i1}^{'} = A_{i1} - S_{i2}; A_{i2}^{'} = A_{i2} - S_{i1} \ \forall i$

Step 3: Check the feasibility of solution, i.e. If $A^{'}_{i1} \geq A^{'}_{j2}$ or $A^{'}_{i1} \leq A^{'}_{j2}$ for all $i, j, i \neq j$**.** If the condition hold then goto step 4 else the proposed algorithm is not applicable.

Step 4: Obtain the job J_1 (say) having maximum processing time on 1st machine.

Step 5: Obtain the job J_n (say) having minimum processing time on 2nd machine.

Step 6: If $J_1 \neq J_n$ then put J_1 on the first position and J_n as the last position & go to step 9, Otherwise go to step 7.

Step 7: Take the difference of processing time of job J_1 on M_1 from job J_2 (say) having next maximum processing time on M_1. Call this difference as G_1.Also, Take the difference of processing time of job J_n on M_2 from job J_{n-1}(say) having next minimum processing time on M_2. Call the difference as G_2.

Step 8: If $G_1 \le G_2$ put J_n on the last position and J_2 on the first position otherwise put J_1 on 1st position and J_{n-1} on the last position.
Step 9: Arrange the remaining (n-2) jobs between 1st job & last job in any order, thereby we get the sequences $S_1, S_2, \ldots, S_r$.
Step10: Compute the total completion time CT (S_k) k=1, 2, …, r.
Step 11: Calculate utilization time U_2 of 2nd machine $U_2 = CT(S_k) - A_{11}(S_k)$; k=1,2, ..., r.
Step 12: Find rental cost $R(S_k) = t_{n1}(S_k) \times C_1 + U_2 \times C_2$, where C_1 & C_2 are the rental cost per unit time of 1st & 2nd machine respectively.

8 Numerical Illustration

Consider 5 jobs, 2 machine flow shop problem with processing time and setup time associated with their respective probabilities as given in the table 2. The rental cost per unit time for machines M_1 and M_2 are 4 units and 10 units respectively. Our objective is to obtain optimal schedule to minimize the total rental cost of the machines, under the rental policy P.

Table 2 Processing time and setup time associated with their respective probabilities

Job	Machine M_1				Machine M_2			
i	a_{i1}	p_{i1}	s_{i1}	q_{i1}	a_{i2}	p_{i2}	s_{i2}	q_{i2}
1	18	0.1	6	0.1	13	0.1	2	0.2
2	12	0.3	7	0.2	8	0.3	4	0.3
3	14	0.3	4	0.3	16	0.1	6	0.2
4	13	0.2	6	0.3	14	0.2	5	0.1
5	25	0.1	4	0.1	6	0.3	4	0.2

Solution: The expected processing and setup times for machines M_1 and M_2 are as shown in table 3.

Table 3 The expected processing and setup times for machines M_1 and M_2

Job	Machine M_1		Machine M_2	
I	A_{i1}	S_{i1}	A_{i2}	S_{i2}
1	1.8	0.6	1.3	0.4
2	3.6	1.4	2.4	1.2
3	4.2	1.2	1.6	1.2
4	2.6	1.8	2.8	0.5
5	2.5	0.4	1.8	0.8

The expected flow times for the two fictious machines M_1 and M_2 are as shown in table 4.

Table 4 The expected flow times for the two fictitious machines G_i and H_i

Job	Machine G_i	Machine H_i
i	A'_{i1}	A'_{i2}
1	1.4	0.7
2	2.4	1.0
3	3.0	0.4
4	2.1	1.0
5	1.7	1.4

Here $A'_{i1} \geq A'_{j2}$ for all i,j. Also, Max A'_{i1} = 3.0 which is for job 3.i.e. J_1= 3.
Min A'_{j2}= 0.4 which is for job 3.i.e. J_n = 3.i.e. J_1=J_n
Therefore G_1= J_1- J_2 = 3.0 -2.4 = 0.6 and G_2 = J_{n-1}- J_n = 0.7 – 0.4 = 0.3.
.i.e. $G_1 \geq G_2$, therefore J_1 = 3rd job will be on 1st position and J_{n-1}= 1st will be on the last position.
Therefore, the optimal sequences are:
S_1 = 3 – 4 – 2 – 5 – 1, S_2 = 3 – 5 – 2 – 4 – 1, S_3 = 3 – 5 – 4 – 2 – 1, ...
The total elapsed time is same for all these possible 6 sequences S_1, S_2, S_3, S_4, S_5,--S_6.
The In- out table for any of these 6 sequences S_1, S_2, S_3, …, S_6; say for
S_1 = 3 – 4 – 2 – 5 – 1 is as shown in table 5.

Table 5 The In- out table for S_1 = 3 – 4 – 2 – 5 – 1

Job	M1	M2
	In- out	In – out
3	0.0 – 4.2	4.2 – 5.8
4	5.4 – 8.0	8.0 – 10.8
2	9.8 – 13.4	13.4 – 15.8
5	14.8 – 17.3	17.3 – 19.1
1	17.7 – 19.5	19.9 – 21.2

Therefore, the total elapsed time =CT(S_1)=21.2 units and
Utilization time for M_2 =$U_2(S_1)$ = 21.2 – 4.2 = 17 units. Also, $t_{n,1}$ = 19.5.
Therefore the total rental cost for each of the sequence (S_k), k = 1, 2, 3, ..., 6 is
R(S_k) = 19.5 × 4 + 17 × 10= 78.0 + 170= 248.0 units

9 Remarks

If we solve the same problem by Johnson's [1] methods we get the optimal sequence as S= 5 – 4 – 2 – 1 – 3. The in-out flow table is as shown in table 6.

Table 6 The in-out flow table for S= 5 – 4 – 2 – 1 – 3

Jobs	Machine M_1	Machine M_2
i	In - Out	In - Out
5	0.0 – 2.5	2.5 – 4.3
4	2.9 – 5.5	5.5 – 8.3
2	7.3 – 10.9	10.9 – 13.3
1	12.3 – 14.1	14.5 – 15.8
3	14.7 – 18.9	18.9 – 20.5

Therefore, the total elapsed time =CT(S) = 20.5 units and
Utilization time for M_2 = $U_2(S)$ = 18.0 units. Also t_{n1}=18.9.
Therefore Rental Cost is R(S) = 255.6 units.

10 Conclusions

The algorithm proposed in this paper for optimal two stage specially structured flow shop scheduling problem in which the processing times and independent setup times each are associated with probabilities is more efficient and less time consuming as compared to the algorithm proposed by Johnson's [1] to find an optimal sequence to minimize the rental cost of machines. Due to our rental policy the ideal time for second machine is always be minimum. Therefore rental cost will always be minimum.

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Appendix

Computer Programme

```
#include<iostream.h>
#include<stdio.h>
#include<conio.h>
#include<process.h>
#include<math.h>

int n;
float a1[16],b1[16],a_1[16],b_1[16],a11[16],b11[16],s11[16],s22[16];
float macha[16],machb[16],macha1[16],maxv,u2;
int j[16],j1[16],j2[16],j3[16];
float costa,costb,cost;

int main()
{
        clrscr();
        int a[16],b[16],s1[16],s2[16];
        float p[16],q[16],u[16],v[16],g1,g2;
        cout<<"How many Jobs (<=15) : ";
        cin>>n;
        if(n<1 || n>15)
        {
                cout<<endl<<"Wrong input, No. of jobs should be less than 15..\n Exitting";
                getch();
                exit(0);
        }
        for(int i=1;i<=n;i++)
                {
                cout<<"\nEnter the processing time, Setup time and their probability of "<<i<<" job for machine A : ";
                cin>>a[i]>>p[i]>>s1[i]>>u[i];
                cout<<"\nEnter the processing time, Setup time and their probability of "<<i<<" job for machine B : ";
                cin>>b[i]>>q[i]>>s2[i]>>v[i];
                //Calculate the expected processing & Setup times of the jobs for the machines:
                a_1[i] = a[i]*p[i];b_1[i] = b[i]*q[i];
                s11[i] = s1[i]*u[i];s22[i]= s2[i]*v[i];
                j[i]=i;
                a1[i]=a_1[i]-s22[i]; b1[i]=b_1[i]-s11[i];
                }
        cout<<"\n Enter the rental cost for Machine M1 & Machine M2 :";
        cin>>costa>>costb;
        cout<<endl<<"Expected processing time of machine A and B: \n";

        for(i=1;i<=n;i++)
```

```
        {
        cout<<"\n"<<j[i]<<"\t"<<a1[i]<<"\t"<<b1[i]<<"\t";
        cout<<endl;
        }
        for(i=1;i<=n;i++)
        {
        if((a1[i]>=b1[i])^(a1[i]<=b1[i]))
          {
                a1[i]=a1[i],b1[i]=b1[i];
          }
        else
        {
        cout<<"\n The data is not in standard form";
        getch();
        exit(0);
        }
        }
void sort(float [],int);// function declaration
for(i=1;i<=n;i++)
{
a11[i]=a1[i];
}
sort(a11,n);//fuction call
cout<<"\nSorted processing times in ascending order of Machine A :\n";

for(i=1;i<=n;i++)
{
j1[i]=j[i];
cout<<"\n"<<j1[i]<<"\t"<<a11[i];
}
for(i=1;i<=n;i++)
{
b11[i]=b1[i];
j[i]=i;
}
sort(b11,n);// function call
cout <<"\nSorted processing times in ascending order of Machine B :\n";
for(i=1;i<=n;i++)
{
j2[i]=j[i];
cout<<"\n"<<j2[i]<<"\t"<<b11[i];
}
if(j1[n]!=j2[1])
{
j3[1]=j1[n];j3[n]=j2[1];
for(int k=2;k<=n-1;k++)
{
if(j1[k-1]!=j2[1])
```

```
{
j3[k]=j1[k-1];
}
else
{
if(j1[n-1]!=j2[1])
{
j3[k]=j1[n-1];
}
}
}
}
else
{
g1=a11[j1[n]]-a11[j1[n-1]];
g2=b11[j2[2]]-b11[j2[1]];
if(g1<=g2)
{
j3[1]=j1[n-1];j3[n]=j2[1];
for(int g=2;g<=n-1;g++)
{
j3[g]=j1[g-1];
}
}
else
{
j3[1]=j1[n];j3[n]=j2[2];
for(int f=2;f<=n-1;f++)
{
j3[f]=j2[f+1];
}
}
}
macha[1]=a_1[j3[1]];machb[1]=macha[1]+b_1[j3[1]];

// displaying solution
cout<<"\n\n\t*********************************************************";
cout<<"\n\t"<<"optimal sequence is";
for(i=1;i<=n;i++)
{
cout<<"\t"<<j3[i];
}
float time =0.0;
cout<<endl<<endl<<"In-Out Table is"<<endl<<endl;
cout<<"Jobs"<<"\t"<<"Machine M1"<<"\t"<<"Machine M2"<<endl;
cout<<j3[1]<<"\t"<<time<<"--"<<macha[1]<<"\t"<<"\t"<<macha[1]<<"--
"<<machb[1]<<"\t"<<endl;
for(i=2;i<=n;i++)
```

```
{
macha1[i]=macha[i-1]+s11[j3[i-1]];
macha[i]=macha1[i]+a_1[j3[i]];
if(machb[i-1]+s22[j3[i-1]]>macha[i])
{
maxv= machb[i-1]+s22[j3[i-1]];
}
else
{
maxv=macha[i];
}
machb[i]=maxv+b_1[j3[i]];
cout<<j3[i]<<"\t"<<macha1[i]<<"--"<<macha[i]<<"\t"<<"\t"<<maxv<<"--
"<<machb[i]<<"\t"<<endl;
}
u2=machb[n]-macha[1];
cost=macha[n]*costa+u2*costb;
cout<<"\n\nThe total rental cost of machines is:"<<cost;
cout<<"\n\n\t***********************************************************";
getch();
return 0;
}
void sort(float x[],int n)// function decleration
{
float temp; int temp1;
//outer for loop to control no of passea
for(int k=1;k<=n;k++)
{
//inner for loop for making comparison per pass
for(int m=1;m<=n-k;m++)
{
        if(x[m]>x[m+1])
                {
        temp=x[m];temp1=j[m];
        x[m]=x[m+1];j[m]=j[m+1];
        x[m+1]=temp;j[m+1]=temp1;
                }
        }
}
}
```