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The Tchebycheff Norm for Ranking DMUs in Cellular Manufacturing Systems with Assignment Worker

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Abstract This paper develops an integer mathematical programming model to design the cellular manufacturing systems under data envelopment analysis. Since workers have an important role in doing jobs on machines, assignment of workers to cells becomes a crucial factor for fully utilization of cellular manufacturing systems (CMS). The aim of the proposed is to minimize backorder costs and intercellular costs. The Data Envelopment Analysis (DEA) is performed to determine the most efficient alternative among alternatives that considered by employing the average machine utilization, the average worker utilization, the number of product as the output variables and the number of machines, the number of workers, the number of parts and demand levels as the input variables. We are using the Tchebycheff norm method to rank the best DMUs.

Keywords Mathematical Programming, Manufacturing System, Data Envelopment Analysis.

1 Introduction

Group technology (GT) is a manufacturing philosophy in which similar parts are identified and grouped together to take advantages of their similarities in manufacturing and design. GT was first proposed by Mitrofanov [1], and was propagated by Burbidge [2], who developed methods suitable for hand computation. Cellular manufacturing (CM) is a successful application of GT concepts. The major advantages of CM have been reported in the literature as reduction in setup time, reduction in throughput time, reduction in work-in-process inventories, reduction in material handling costs, better quality and production control, increment in flexibility, etc. (Heragu [3], Wemmerlov and Hyer [4]. One of the key issues encountered in the implementation of a CMS is the cell formation problem (CFP). In the past several years, many solution methods have been developed for solving cell formation problem (CFP) by a binary machine-part incidence (two-dimensional) matrix. Some comprehensive summaries and taxonomies considering the CFP as a machine-part incidence matrix include Singh [5], Offodile et al. [6], Selim et al. [7] and Mansouri et al. [8]. Moreover, recently some approaches that have been developed to the two-dimensional CFP are: genetic algorithms (Goncalves and Resende [9] and Mahdavi et al. [10]), tabu search (Lozano et al. [11] and Wu et al. [12]), neural network (Soleymanpour et al. [13]), mathematical programming (Albadawi

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et al. [14] and Mahdavi et al. [15]), simulated annealing (Wu et al. [16] and Pailla et al. [17]) and similarity coefficients-based method (Yin and Yasuda [18] and Oliveira et al. [19]).

One of the main points in CM is considering human issues since ignoring this factor can considerably reduce benefits of the utility of the cell manufacturing. In some of the previous research papers this issue is discussed. Nembhard [20] described a greedy heuristic approach based on individual learning rate for the improvement of productivity in organizations through targeted assignment of workers to tasks. Norman et al. [21] proposed a mixed integer programming model for assigning workers to manufacturing cells in order to maximize the profit. Bidanda et al. [22] presented an overview and evaluation of the diverse range of human issues involved in CM based on an extensive literature review. In Wirojanagud et al. [23] a workforce planning model that incorporates individual worker differences in ability to learn new skills and perform tasks was presented. The model allows a number of different staffing decisions (i.e., hire and fire) in order to minimize workforce related and missed production costs. Aryanezhad et al. [24] presented a new model to deal with dynamic cell formation and worker assignment problem with considering part routing flexibility and machine flexibility and also promotion of workers from one skill level.

Min and Shin [25] created a prototype of three-dimensional GT. Their method was to insert the third factor, operator, into the sorted incidence matrix of parts and machines. Parkin and Li [26] proposed an algorithm for N-dimensional GT. Their algorithm focused on each incidence matrix, sorting each separately. Li [27] showed a method of solving multi-dimensional GT problem. Mahdavi et al. [10] presented a new mathematical model to minimize the number of voids and exceptional elements in a three dimensional (cubic) machine-part-worker incidence matrix. One important aspect of the cell formation problem is its efficiency measurement procedures. Besides, there are few researches on the efficiency measurement of the cell formation. Especially, very little CMS research has been directed at human factor issues (Scott et al., [28]). Ertay and Ruan [29] took advantage of the cross-efficiency evaluation to determine the best labor assignment in CMS. They study concentrates on efficiency measurement and the determination of the number of operators in CMS when the demand rate and the transfer batch size as a rate of batch size change. Both the inputs and outputs of their study were procured by means of simulation of CMS.

In this paper we develop an integer mathematical programming to design the CMS, by means of considering several situation for each of input variables, the number of machines, the number of workers, the number of parts and demand levels, we get several different alternatives to decision maker. To determine the most efficient alternative, for each alternative we use the developed CMS model to gain the average machine utilization, the average worker utilization, and mean of product as the output variables of the alternative and then DEA performed to determine the most efficient scenario among all the scenarios that considered.

2 Data envelopment analysis methodology

Data envelopment analysis (DEA), proposed by Charnes et al. [30] is a mathematical programming technique that measures the relative efficiency of decision making units (DMUs) with multiple inputs and outputs but with no obvious production function to aggregate the data in its entirety. In most models of DEA (such as CCR), the best performers have efficiency score unity, and, from experience, we know that usually there are plural DMUs which have this ‘‘efficient status’’. To discriminate between these efficient DMUs is an interesting research subject. Ranking DMUs is one of the main problems in DEA. Several authors have proposed methods for ranking the best performers. See for example Adler et al. [31].

Recently, several authors have proposed some methods based on norms. Jahanshahloo et al. [32] introduced L_1-norm approach and Rezai balf et al. [33] presented ranking model L_∞-norm (or Tchebycheff norm) in data envelopment analysis. In this paper, we are use the ranking method based on the tchebycheff norm proposed by Rezai balf et al. [33] that it seems to have superiority over other existing methods, because this method is able to remove the existing deficiencies in some methods, such as Anderson and Peterson [34] that it is sometimes infeasible. The L_∞-norm model always is feasible.

2.1 Background DEA

2.1.1 DEA model

DEA is a mathematical model that measures the relative efficiency of decision making units (DMUs) with multiple inputs and outputs but with no obvious production function to aggregate the data in its entirety. By comparing *n* units with *s* outputs denoted by $y_{rj}\,(r=1,...,s)$, and m inputs denoted by $x_{ij}\,(i=1,...,m)$, that all of them are non-negative and each DMU has at least one strictly positive input and output. The efficiency of a specific DMU_P can be evaluated by the CCR model (Charnes et al. [30]), of DEA as follows:

$$
\begin{aligned}
& Max \quad \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}} \\
& s.t. \\
& \qquad \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \le 0, \quad j=1,...,n, \qquad (1) \\
& \qquad u_r \ge 0, \quad r=1,...,s, \\
& \qquad v_i \ge 0, \quad i=1,...,m.
\end{aligned}
$$

where $u_r\,(r=1,...,s)$, and $v_i\,(\mathrm{i}=1,...,m)$, represent the output and input weights, respectively.

Besides, the fractional program is not used for actual computation of the efficiency scores due to its non-convex and nonlinear properties. Hence, by using Charnes and Cooper [35] transformation, model 1 can be equivalently transformed into the linear program below for solution:

$$
\begin{aligned}
& Max \sum_{r=1}^{s} u_r y_{ro} \\
& s.t. \\
& \qquad \sum_{i=1}^{m} v_i x_{io} = 1 \\
& \qquad \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0, \quad j = 1,\dots,n, \\
& \qquad u_r \ge 0, \qquad r = 1,\dots,s, \\
& \qquad v_i \ge 0, \qquad i = 1,\dots,m.
\end{aligned} \tag{2}
$$

2.1.2 L_∞-norm in DEA

By comparing n units with s outputs denoted by $y_{rj}, r = 1,\dots,s,$ and m inputs denoted by $x_{ij}, i = 1,\dots,m$, that all of them are non-negative and each DMU has at least one strictly positive input and output. The production possibility sets (PPS) is defined as:

$$T_c = \{(X,Y) \mid X \ge \sum_{j=1}^{n} \lambda_j X_j, Y \le \sum_{j=1}^{n} \lambda_j Y_j, \lambda_j \ge 0, j = 1,\dots,n\} \tag{3}$$

Rezai balf et al. [33] introduced ranking model L_∞-norm in data envelopment analysis. They assumed that the DMU_O is extremely efficient. By omitting (X_O,Y_O) from T_C, they defined the production possibility set T'_C as:

$$T_c' = \{(X,Y) \mid X \ge \sum_{j=1, j\ne o}^{n} \lambda_j X_j, Y \le \sum_{j=1, j\ne o}^{n} \lambda_j Y_j, \lambda_j \ge 0, j = 1,\dots,n\} \tag{4}$$

They consider the following model to obtain the ranking score of DMU_O:

$$
\begin{aligned}
& Min \;\; \Phi_c^o(X,Y) = Max \left(\left\{ \left| x_{io} - \sum_{j=1, j\ne o}^{n} \lambda_j x_{ij} \right| \right\}_{i=1,\dots,m}, \left\{ \left| y_{ro} - \sum_{j=1, j\ne o}^{n} \lambda_j y_{rj} \right| \right\}_{r=1,\dots,s} \right) \\
& s.t. \\
& \qquad \sum_{j=1, j\ne o}^{n} \lambda_j x_{ij} \ge x_{io}, \quad i = 1,\dots,m, \\
& \qquad \sum_{j=1, j\ne o}^{n} \lambda_j y_{rj} \le y_{ro}, \quad r = 1,\dots,s, \\
& \qquad \lambda_j \ge 0, \qquad\qquad j = 1,\dots,n.
\end{aligned} \tag{5}
$$

where $X = (x_1,\dots,x_m)$ $,Y = (y_1,\dots,y_s)$ and $\Lambda = (\lambda_1,\dots,\lambda_{o-1},\lambda_{o+1},\dots,\lambda_n)$ are the variables of the model 5 and $\Phi_c^o(X,Y)$ is a distance (X_o,Y_o) from (X,Y) by using L_∞-norm. It is obvious that the model 5 is non-linear. In order to converting this model to a linear form, the set T_c'' is defined as:

$T_c'' = T_c' \bigcap \{(X,Y) | X \geq X_o, Y \leq Y_o\}$.

Therefore, by added the constrains $X \geq X_o$ and $Y \leq Y_o$ to the model 5 they obtained the linear form as follows:

$$
\begin{aligned}
& Min \quad \varphi_o \\
& s.t. \\
& \quad \varphi_o \geq \sum_{j=1, j\neq o}^{n} \lambda_j x_{ij} - x_{io}, \qquad i = 1,\dots,m, \\
& \quad \varphi_o \geq y_{ro} - \sum_{j=1, j\neq o}^{n} \lambda_j y_{rj}, \qquad r = 1,\dots,s, \\
& \quad \lambda_j \geq 0, \qquad\qquad\qquad j = 1,\dots,n.
\end{aligned} \tag{6}
$$

where $\varphi_o = Max\left(\left\{\sum_{j=1, j\neq o}^{n} \lambda_j x_{ij} - x_{io}\right\}_{i=1,\dots,m}, \left\{y_{ro} - \sum_{j=1, j\neq o}^{n} \lambda_j y_{rj}\right\}_{r=1,\dots,s}\right)$.

Theorem 1. Suppose $(X_o, Y_o) \in T_c$ is extreme efficient. For each $(\bar{X}, \bar{Y}) \in T_c' \setminus T_c''$ there exists at least a member of T_c'', say $(\tilde{X}, \tilde{Y})$, such that $\Phi_c^o(\tilde{X}, \tilde{Y}) \leq \Phi_c^o(\bar{X}, \bar{Y})$.

Theorem 2. In any optimal solution the model 6, at least one of inputs (outputs) constraints is active.

Theorem 3. The projected point of DMU_O in model 6 lies on the efficient frontier.

Theorem 4. Model 6 is always feasible and bounded.

3 Problem formulation

In this section, the mathematical model has been presented based on CMS with worker flexibility under following assumptions:

- The processing time for all operations of a part type on different machine types are known and deterministic.
- The demand for each part type is known and deterministic.
- The capacity of each machine type is known
- The available time of each worker is known
- The number of production for each part littler than the number of demand for each part.

3.1 Indices and their upper bounds

P Number of part types
W Number of worker types
M Number of machine types

C	Number of cells
I	Index for part type (i=1,2,…P)
W	Index for worker (w=1, 2,…W)
M	Index for machine type (m=1, 2,…M)
K	Index for cell (k=1,2,…C)

3.2 Input parameters

r_{imw}	1 if machine type m is able to process part i with worker w ; = 0 otherwise
a_{im}	1 if part i needs machine type m; = 0 otherwise
LM_k	Minimum size of cell k in terms of the number of machine types
LP_k	Minimum size of cell k in terms of the number of parts
LW_k	Minimum size of cell k in terms of the number of workers
RW_w	Available time for worker w
RM_m	Available time for machine m
t_{imw}	Processing time of part i on machine type m with worker w
D_i	Demand of part i
ε_i	Unit backorder cost of part i
α_i	Unit cost of intercell movement
A	An arbitrary big positive number

3.3 Decision Variables

x_{mk}	1 if machine type m is assigned for cell k; =0 otherwise
y_{ik}	1 if part i is assigned to cell k; =0 otherwise
z_{wk}	1 if worker w is assigned for cell k; =0 otherwise
d_{imwk}	1 if part i is to be processed on machine type m with worker w in cell; =0 otherwise
P_i	Number of part i to be produced

3.4 Mathematical formulation
3.4.1 Objective functions

$$Min = \sum_{i=1}^{P} \alpha_i P_i \left[\begin{array}{ll} \sum_{k=1}^{C}\sum_{m=1}^{M}\sum_{w=1}^{W}[y_{ik}x_{mk}(1-z_{wk})d_{imwk}] & (5.1) \\ +\sum_{k=1}^{C}\sum_{m=1}^{M}\sum_{w=1}^{W}[2\times x_{mk}(1-y_{ik})(1-z_{wk})d_{imwk}] & (5.2) \\ +\sum_{k=1}^{C}\sum_{m=1}^{M}\sum_{w=1}^{W}[x_{mk}(1-y_{ik})z_{wk}d_{imwk}] & (5.3) \end{array} \right]$$

$$+\sum_{i=1}^{P} \varepsilon_i [D_i - P_i] \qquad (5.4)$$

3.4.2 Contraints

$$\sum_{k=1}^{C}\sum_{m=1}^{M}\sum_{i=1}^{P} d_{imwk} t_{imw} P_i \le RW_w \qquad \forall w; \qquad (6)$$

$$\sum_{w=1}^{W}\sum_{i=1}^{P} d_{imwk} t_{imw} P_i \le RM_m \qquad \forall m,k; \qquad (7)$$

$$D_i \ge P_i \qquad \forall i; \qquad (8)$$

$$d_{imwk} \le r_{imw} x_{mk} \qquad \forall i,m,w,k; \qquad (9)$$

$$\sum_{k=1}^{C}\sum_{w=1}^{W} d_{imwk} = a_{im} \qquad \forall i,m; \qquad (10)$$

$$\sum_{k=1}^{C} y_{ik} = 1 \qquad \forall i; \qquad (11)$$

$$\sum_{i=1}^{P} y_{ik} \ge LP_K \qquad \forall k; \qquad (12)$$

$$\sum_{k=1}^{C} x_{mk} = 1 \qquad \forall m; \qquad (13)$$

$$\sum_{m=1}^{M} x_{mk} \ge LM_k \qquad \forall k; \qquad (14)$$

$$\sum_{k=1}^{C} z_{wk} = 1 \qquad \forall w; \qquad (15)$$

$$\sum_{w=1}^{W} z_{wk} \ge LW_k \qquad \forall k; \qquad (16)$$

$$x_{mk}, y_{ik}, z_{wk}, d_{imwk} \in \{0,1\} \qquad \forall i,m,w,k; \qquad (17)$$

$$P_i \ge 0 \qquad \forall i; \qquad (18)$$

The objective function consists of several costs items as follows:

(1), (2), (3) *Exceptional Elements*: The first, second and third terms is to minimize the total number of exceptional elements in machine-part-worker incidence matrix. The numbers of exceptional elements for parts are calculated based on the status of availability of corresponding machine and worker as shown in Table 1. If the corresponding machine and worker both are not in the cell, the number of exceptional elements will take value 1 or 2 depending on the availability of machine and worker in one cell or at different cells, respectively. The equations (5.1)-(5.3) can be simplified as follows:

$$\sum_{i=1}^{P}\sum_{k=1}^{C}\sum_{m=1}^{M}\sum_{w=1}^{W}\left[x_{mk}(2-y_{ik}-z_{wk})d_{imwk}\right]$$

Table 1 Status of exceptional elements

Case	Part	Machine	Worker	Exceptional Elements
1	✓	✓	✓	0
2	✓	✓	×	1
3	×	✓	×	2
4	×	✓	✓	1

Note: '✓ ' denotes included and '×'excluded.

To clarify the calculation of exceptional elements, this concept is discussed in Figures 1, 3 and 4. In Figure 1, part type 3 needs worker 5 to get processed on machine type 2. However, part type 3 and machine type 2 have been assigned to cell 3 while worker 5 is in cell 2. Thus, worker 5 has to come to cell 3 which implies one intercellular movement (case 2 of Table 1).

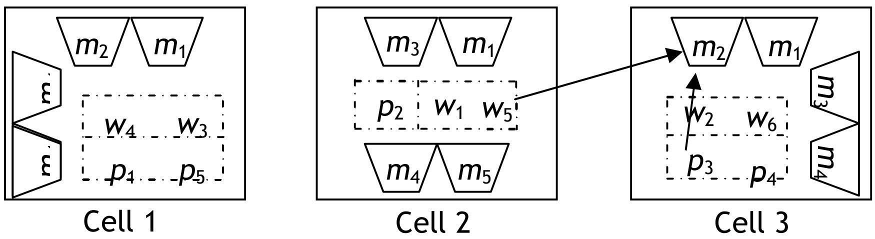

Fig. 1 Exceptional element for case 2 in Table 1

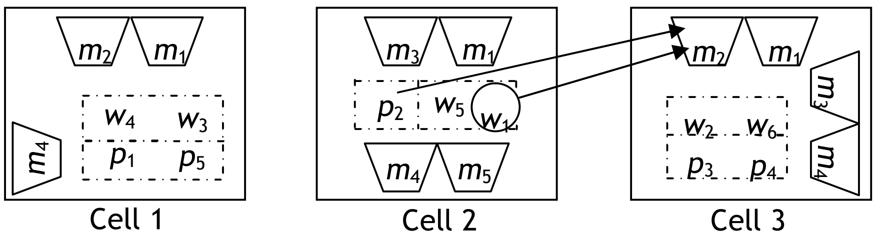

Fig. 2 Exceptional element for case 3 in Table 1

In Figure 2, let us discuss case 3 of Table 1. In this figure, suppose machine type 2 and worker 1 are required to process part type 2. Furthermore, part type 2, machine type 2 and worker 1 have been assigned to cell 2, cells 1 and 3, and cell 2, respectively. Since part type 2 and worker 1 have to move to cell 3, the number of exceptional elements will be 2.

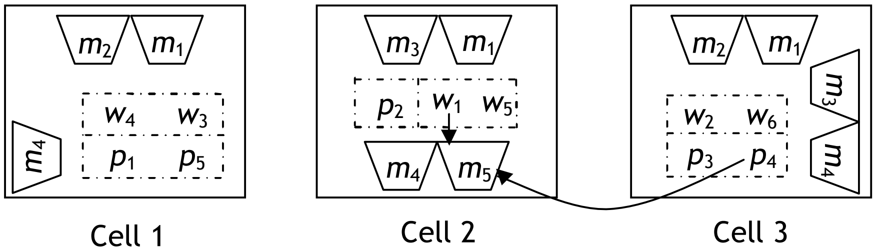

Fig. 3 Exceptional element for case 4 in Table 1

Case 4 of Table 1 is demonstrated in Figure 3. In this figure, suppose machine type 5 and worker 1 are required to process part type 4. Furthermore, suppose part type 4 has been assigned to cell 3 while machine type 5 and worker 1 are in cell 2. Therefore, part type 4 has to move to cell 2 which results in one intercellular movement in this case.

(4) ***Backorder cost***: The cost of delay in delivery of all parts. This item is calculated the number of demand for each part, minus the number of production for each part, multiply by the unit backorder cost each part.

3.5 Description of constraints

Constraints (6) and (7) ensure that the available time for workers and capacity of machines are not exceeded. Constraint (8) the number of production for each part smaller than the number of demand for each part. Constraint (9) ensures that when machine type m is not in cell k, then d_{imwk}=0. Equation (10) implies that only one worker is allotted for processing each part type on each machine type in one cell. This model is flexible for doing same job with different workers. This means that if one part type is required to be processed by one machine type; more than one worker would be able to service this machine type. Equation (11) ensures that each part type is assigned to only one cell. Constraint (12) forces the lower bound for the number of parts to be allocated to each cell. Equation (13) guarantees that each machine type is assigned to only one cell. Constraint (14) prevents from assigning less than LM_k machines to cell k. Equation (15) guarantees that each worker will be assigned to only one cell. Constraint (16) ensures that at least LW_k workers will be assigned to cell k in each period;

3.6 Linearization of the proposed model

In this section, an attempt is made to linearize the objective function of the mathematical model proposed in Section 3.4.

Procedure

The linearization procedure that we propose here consists of two steps that are given by the two lemmas stated below. The non-linear terms in the objective function and constraints (6), (7) are multiplication of binary and integer variables which can be linearized using the following auxiliary integer variables E_{imwk}, F_{imwk}, S_{imwk}, and G_{imwk}. Each lemma for linearization is followed by a proof that illustrates the meaning of each auxiliary (linearization) variable and the expressions where they are used.

Lemma1. The non-linear terms in the objective function and constraints (6) and (7) of the mathematical model can be linearized with $E_{imwk} = P_i . d_{imwk}$, and $F_{imwk} = y_{mk} . E_{imwk}$ and $S_{imwk} = z_{wk} . E_{imwk}$ under the following sets of constraints:

$$E_{imwk} \le P_i + A(1-d_{imwk}) \qquad \forall i, m, w, k; \tag{19.1}$$
$$E_{imwk} \ge P_i - A(1-d_{imwk}) \qquad \forall i, m, w, k; \tag{19.2}$$
$$E_{imwk} \le A.d_{imwk} \qquad \forall i, m, w, k; \tag{19.3}$$

and

$$F_{imwk} \le E_{imwk\, i} + A(1-y_{mk}) \qquad \forall i, m, w, k; \tag{19.4}$$
$$F_{imwk} \ge E_{imwk\, i} - A(1-y_{mk}) \qquad \forall i, m, w, k; \tag{19.5}$$
$$F_{imwk} \le A.y_{mk} \qquad \forall i, m, w, k; \tag{19.6}$$

and

$$S_{imwk} \le E_{imwk\, i} + A(1-z_{wk}) \qquad \forall i, m, w, k; \tag{19.7}$$
$$S_{imwk} \ge E_{imwk\, i} - A(1-z_{wk}) \qquad \forall i, m, w, k; \tag{19.8}$$
$$S_{imwk} \le A.z_{wk} \qquad \forall i, m, w, k; \tag{19.9}$$

Proof. This can be shown for each of the two possible cases that can arise.

(i) $d_{imwk} . P_i = P_i$. $\qquad \forall i, m, w, k;$

Such a situation arises when $d_{imwk} = 1$ so, constraints (19.1) and (19.2) implies $E_{imwk} \le P_i$ and $E_{imwk} \ge P_i$ and ensures that $E_{imwk} = P_i$.

(ii) $d_{imwk} . P_i = 0$. Such a situation arises under one of the following three sub-cases:
(a) $d_{imwk} = 1$ and $P_i = 0$. $\qquad \forall i, m, w, k;$
(b) $d_{imwk} = 0$ and $P_i > 0$. $\qquad \forall i, m, w, k;$
(c) $d_{imwk} = 0$ and $P_i = 0$. $\qquad \forall i, m, w, k;$

In all of the three sub-cases given above, E_{imwk} takes the value of 0, because in these cases, constraint (19.3) implies $E_{imwk} \le 0$ and ensures that $E_{imwk} = 0$. Because E_{imwk} has not a strictly positive cost coefficient, the minimizing objective function doesn't ensures that $E_{imwk} = 0$. Thus, constraint (19.3) should be added to the mathematical model.
The performance of constraints (19.4) - (19.9) is similar to constraints' (19.1) and (19.3).

Lemma2. The non-linear terms in the objective function can be linearized with $G_{imwk} = x_{ik} . E_{imwk}$, under the following set of constraints:

$$G_{imwk} \ge E_{imwk} - A(1-x_{ik}) \qquad \forall i, m, w, k; \tag{20}$$

Proof. Consider the following two cases:
(i) $x_{ik} \,.\, E_{imwk} = 0$. Such a situation arises under one of the following three sub-cases:
(a) $x_{ik} = 1$ and $E_{imwk} = 0$. $\forall i, m, w, k;$
(b) $x_{ik} = 0$ and $E_{imwk} > 0$. $\forall i, m, w, k;$
(c) $x_{ik} = 0$ and $E_{imwk} = 0$. $\forall i, m, w, k;$

In all of the three sub-cases given above, the value of $G_{imwk} = 0$, because in these cases, constraint (20) implies $G_{imwk} \geq 0$ or $-\infty$ and since G_{imwk} has a strictly positive cost coefficient, the minimizing objective function ensures that $G_{imwk} = 0$.

(ii) $x_{ik} \,.\, E_{imwk} = E_{imwk} > 0$. $\forall i, j;$

Such a situation arises when $x_{ik} = 1$ and $E_{imwk} > 0$ so, constraint (20) implies $G_{imwk} \geq E_{imwk}$ and since G_{imwk} has a strictly positive cost coefficient, the minimizing objective function ensures that $G_{imwk} = E_{imwk}$.

3.6.1 The linearized model

The new version of the first, second and third terms of objective function based on new variables, the linear mathematical model becomes as follows:

$$Min = \sum_{i=1}^{P}\sum_{k=1}^{C}\sum_{m=1}^{M}\sum_{w=1}^{W}\alpha_i\left[2\times G_{imwk} - F_{imwk} - S_{imwk}\right]$$

$$E_{imwk}, F_{imwk}, G_{imwk}, S_{imwk} \geq 0 \qquad \forall i, m, w, k; \tag{21}$$

Subject to constraints (8) – (21) and new version of constraints (6) and (7):

$$\sum_{k=1}^{C}\sum_{m=1}^{M}\sum_{i=1}^{P} E_{imwk} t_{imw} \leq RW_w \qquad \forall w; \tag{22}$$

$$\sum_{w=1}^{W}\sum_{i=1}^{P} E_{imwk} t_{imw} \leq RM_m \qquad \forall m, k; \tag{23}$$

4 Using data envelopment analysis in the CMS model
4.1 Choosing the inputs and outputs for DEA model

Manned cells are a very flexible system that can adapt to changes in the customers demand or changes quite easily and rapidly in the product design. The cells described in this study are designed for flexibility. In this study, the DEA is applied to the problem of comparing and evaluating the alternative rescores assignment in a CMS environment. In general, in a number of previous DEA evaluation models, the criteria that are to be minimized are viewed as inputs, and the criteria to be maximized are considered as outputs (Doyle and Green [36]. In other words, usually the DEA assumes that outputs are increasing and more of an output is better than less of the output. Alternatives consisted of reducing the number of machines, the number of workers, the number of parts and the demand of each part in the cell is as follows in details:

- Choice 4 machine among 5 machine type that have different availability level the DM have 4 alternatives to choice machines 1, 2, 3, 4 (alternative A), 1, 2, 3, 5 (B), 1, 2, 4, 5 (C), 1, 3, 4, 5 (D).
- Use 3 worker in the cell that the workers number 1 and number 2 have the same level and the workers number 3 and number 4 have the same level, since the DM have 2 alternative for choose the workers, worker 1, worker 2 and worker 3 (F), worker 2, worker 3 and worker 4 (G).
- The number of parts type for produce is 4 or 5. The production value (backorder cost) all of the parts are the same level so DM have 2 alternatives to choice parts.
- Numbers of demand for each part are 300 and 350, and DM have 2 alternatives to choice volume of the parts.

So DM have $4\times 2\times 2\times 2=32$ alternatives to decision.

To illustrate the capability of the proposed model an alternative have been solved by branch and bound (B&B) method under Lingo 9.0 software package.

In all alternatives we consider two cells with different machines, parts and workers. The data set related to the all alternatives are shown in Tables 2 and 3. Table 2 indicates machines requirement of parts. For example, part type 3 requires machine types 2 and 4. Table 3 indicates capabilities of workers in working with different machines. For example, worker 3 is able to work with machine types 2 and 4. The available time of worker in each period is 20 hours and the available time of machine in each period is 20 hours. Also the processing time is presented in Table 4. Moreover, backorder cost per unit each part types are 1. The number of batch size each part is 100. Also, the minimum size of each cell in terms of the number of machines, parts and workers has been considered to value one.

Table 2 The input data of machine-part incidence matrix

Parts	Machines 1	2	3	4	5
1	1	1	1	1	1
2	1	0	1	0	1
3	0	1	0	1	0
4	0	1	0	1	0
5	1	0	1	0	1

Table 3 The input data of machine-worker incidence matrix

Machines	Workers 1	2	3	4
1	1	0	0	1
2	0	1	1	0
3	1	0	0	1
4	0	1	1	0
5	1	0	0	1

Table 4 The processing time (hrs.)

	Part1				Part2				Part3				Part4				Part5			
	W1	W2	W3	W4	W1	W2	W3	W4	W1	W2	W3	W4	W1	W2	W3	W4	W1	W2	W3	W4
M1	.02			.02	.02			.02									.02			.02
M2		.04	.04							.04	.04			.04	.04					
M3	.03			.03	.03			.03									.03			.03
M4		.01	.01							.01	.01			.01	.01					
M5	.01			.01	.01			.01									.01			.01

Tables 5 shows the results of alternative 1. It indicates the assignment of parts, machines and workers in cells. For instance, workers 3 and 4 are assigned in cell 1, and worker 1 and 2 is assigned in cell 2. Also machine type 2 is assigned in cell 1 and machines 1, 3 and 4 are assigned in cell 2. Moreover, it shows the allotment of worker for each part, in cell for work on corresponding machine. For instance, part 3 shall process with machine 2 (see Table 4) and workers 2 and 3 capability of working to this machine (see Table 5) which this operation is executed by worker 3 in cell 1 (see Table 5).

The volume of products and objective function value including backorder cost and number of exceptional elements (EEs) has been indicated in Table 5. As can be seen, the demand of part 2 in is 350 but this part is 316 produced. This means, the 34 volume of demand of part 2 is which causes backorder cost.

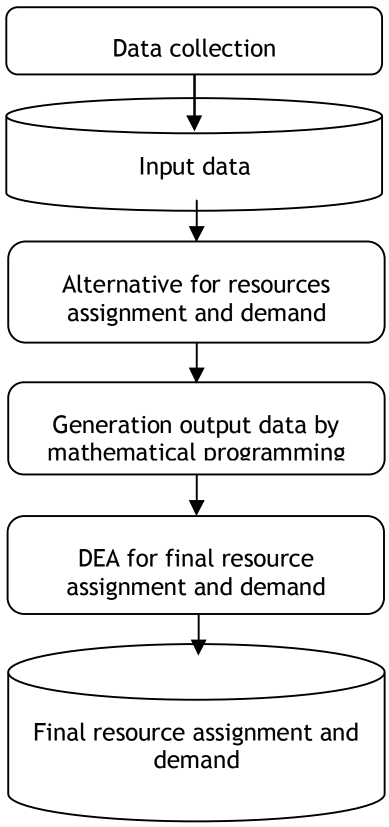

Fig. 4 Steps of the proposed methodology

These alternatives inputs data of the mathematical model are indicated in the Table 6. Steps of the proposed methodology are presented in Fig. 4.

Table 5 The result of alternative 1

Backorder cost		EEs		Part 1				Part 2				Part 3				Part 4				Part 5			
584		14		Workers																			
	Cell1	Cell2	Machine	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
Part	1	2,3,4,5	1					2												2			
Machine	2	1,3,4	2											1[b]				1[b]					
Worker	3,4	1,2	3								2[a]												2[a]
			4										2				2						
Volume of Product				0				316				150				350				350			

[a] The worker movement between cells [b] The part movement between cells

For each alternative we use the developed CMS and calculate average machine utilization (ATUM) and average worker utilization (ATUW) and mean of product from the solutions and set them as the alternative results data of the mathematical model, Table 9 shows the results of 60 alternatives.

Table 6 The inputs of the DEA model

DMU	Number of machines	Number of worker	Number of parts	Demand level	DMU	Number of machines	Number of workers	Number of parts	Demand level
1	4(A)	4	5	350	31	4(C)	3(F)	4	350
2	4(A)	4	5	300	32	4(C)	3(F)	4	300
3	4(A)	4	4	350	33	4(C)	3(G)	5	350
4	4(A)	4	4	300	34	4(C)	3(G)	5	300
5	4(A)	3(F)	5	350	35	4(C)	3(G)	4	350
6	4(A)	3(F)	5	300	36	4(C)	3(G)	4	300
7	4(A)	3(F)	4	350	37	4(D)	4	5	350
8	4(A)	3(F)	4	300	38	4(D)	4	5	300
9	4(A)	3(G)	5	350	39	4(D)	4	4	350
10	4(A)	3(G)	5	300	40	4(D)	4	4	300
11	4(A)	3(G)	4	350	41	4(D)	3(F)	5	350
12	4(A)	3(G)	4	300	42	4(D)	3(F)	5	300
13	4(B)	4	5	350	43	4(D)	3(F)	4	350
14	4(B)	4	5	300	44	4(D)	3(F)	4	300
15	4(B)	4	4	350	45	4(D)	3(G)	5	350
16	4(B)	4	4	300	46	4(D)	3(G)	5	300
17	4(B)	3(F)	5	350	47	4(D)	3(G)	4	350
18	4(B)	3(F)	5	300	48	4(D)	3(G)	4	300
19	4(B)	3(F)	4	350	49	4(E)	4	5	350
20	4(B)	3(F)	4	300	50	4(E)	4	5	300
21	4(B)	3(G)	5	350	51	4(E)	4	4	350
22	4(B)	3(G)	5	300	52	4(E)	4	4	300
23	4(B)	3(G)	4	350	53	4(E)	3(F)	5	350
24	4(B)	3(G)	4	300	54	4(E)	3(F)	5	300
25	4(C)	4	5	350	55	4(E)	3(F)	4	350
26	4(C)	4	5	300	56	4(E)	3(F)	4	300
27	4(C)	4	4	350	57	4(E)	3(G)	5	350
28	4(C)	4	4	300	58	4(E)	3(G)	5	300
29	4(C)	3(F)	5	350	59	4(E)	3(G)	4	350
30	4(C)	3(F)	5	300	60	4(E)	3(G)	4	300

Table 7 The outputs of the DEA model

DMU	Mean of product(%)	ATUM(%)	ATUW(%)	DMU	Mean of product(%)	ATUM(%)	ATUW(%)
1	66.5	72.9	72	31	83.1	51.4	66.3
2	73.3	72.9	72	32	91.6	53.6	75.8
3	83.1	72.9	72	33	56	37	74.1
4	91.6	68.5	60	34	58.3	49.2	66.3
5	51.4	56.9	75.8	35	78.5	38.5	68.5
6	60	56.9	75.8	36	83.3	47	63.6
7	64.2	56.9	75.8	37	78	59.2	59.2
8	75	58.5	70	38	77.3	58.5	57
9	60.8	58.5	77.4	39	97.4	58.5	58.5
10	66.6	62.5	83.6	40	100	52.9	55.8
11	76	66.3	88.5	41	58.8	33.6	44.1
12	83.3	62.9	83.6	42	84.3	58.5	77
13	66.2	59.2	59.2	43	73.7	33.6	44.1
14	73.3	55.8	55.8	44	77.6	26.3	43.6
15	83.2	74.1	74.1	45	78	47	83.6
16	91.6	70	70	46	84.3	59.2	74.1
17	47.6	49.2	66.3	47	97.4	58.5	78.5
18	55.5	49.2	66.3	48	100	52.9	70
19	59.4	49.2	66.3	49	66.5	54.1	64.1
20	69.4	49.2	66.3	50	73.2	64.1	64.1
21	67.7	74.1	99.2	51	83.2	64.1	64.1
22	73.3	70	93.6	52	91.6	61.4	58.5
23	83.2	74.1	99.2	53	56.5	55.8	74.1

DMU	Mean of product(%)	ATUM(%)	ATUW(%)	DMU	Mean of product(%)	ATUM(%)	ATUW(%)
24	91.6	70	93.6	54	66.6	56.3	75.8
25	68.5	53.6	44.3	55	71.4	56.3	75.8
26	73.3	65.8	65.8	56	83.3	51.4	75.8
27	85.7	57	57	57	60.8	58.5	77
28	91.6	64.1	56.3	58	66.6	55.8	73.6
29	66.5	56.3	74.1	59	76.8	54.1	77
30	73.3	49.2	71.4	60	83.3	55.8	73.6

4.2. The most efficient alternative

We used the model 2 for 60 inputs and outputs shown in the Tables 6 and 7, The DEA is applied to the data set of 60 DMU_s. The efficiency scores obtained using DEA are listed in Table 8. The DEA results denote that 9 cases of 60 DMU_s are relatively efficient; however, a ranking cannot be obtained for these DMU_s. Since the efficiencies evaluate 9 of the 60 DMU_s as efficient and cannot discriminate among them any further, a ranking method is needed. We are use the L_∞-norm model 6 to rank these 9 alternatives. The results are shown in Table 9. According to the L_∞-norm method in Table 9, DMU24 is the most efficient alternative, whereas DMU48 is the second most efficient followed by DMU23, DMU47 and others.

Table 8 Efficiency scores that are obtained by DEA

DMU	Efficiency score	DMU	Efficiency score	DMU	Efficiency score	DMU	Efficiency score
1	0.9838	16	1.0000	31	0.8591	46	0.8999
2	0.9836	17	0.6683	32	0.9493	47	1.0000
3	0.9891	18	0.7083	33	0.7470	48	1.0000
4	0.9906	19	0.6847	34	0.7083	49	0.8650
5	0.7679	20	0.7445	35	0.8217	50	0.8702
6	0.8129	21	1.0000	36	0.8481	51	0.9128
7	0.7692	22	0.9446	37	0.8489	52	0.9466
8	0.8357	23	1.0000	38	0.8403	53	0.7530
9	0.7895	24	1.0000	39	0.9874	54	0.7771
10	0.8932	25	0.7587	40	1.0000	55	0.7978
11	0.9014	26	0.8879	41	0.5973	56	0.8813
12	0.9051	27	0.8862	42	0.8972	57	0.7894
13	0.7989	28	1.0000	43	0.7370	58	0.7698
14	0.7988	29	0.7738	44	0.7760	59	0.8339
15	1.0000	30	0.7902	45	0.8767	60	0.8772

Table 9 The Tchebycheff values and Ranking efficient DMUs DEA

DMU	Tch. norm		DMU	Tch. norm		DMU	Tch. Norm	
	Value	Rank		Value	Rank		Value	Rank
15	1.8855e-009	5	23	0.2295	3	40	7.2731e-015	8
16	8.0071e-010	7	24	6.7066	1	47	0.0131	4
21	1.4388e-009	6	28	3.4715e-017	9	48	1.9761	2

5 Conclusions

In this paper, a neutral DEA in CMS is proposed for a three-dimensional machine-part-worker incidence matrix which demonstrates a cubic representation of assignment in cellular manufacturing system. Moreover, the new concept of exceptional elements is discussed to show the interpretation of inter-cell movements of both workers and parts for processing on corresponding machines. The proposed approach minimizes backorder cost and intercellular cost in a cellular manufacturing system. The DEA approach performed for determining the most efficient alternative among 60 alternatives that considered. As a result of the application of classic DEA model, 9 alternatives are determined as relatively efficient. To increase discriminating power among alternatives and ranking, the tchebycheff-norm ranking method was employed.

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