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# Efficiency Measurement of Bank Branches with Undesirable Output Using Non-Radial Models of Data Envelopment Analysis

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**Abstract** Today, banks, as intermediaries, seek to attract and allocate resources to obtain greater benefit. Therefore, the bank assessment is considered as an important subject in the banking industry. Data envelopment analysis is an effective technique in evaluating the efficiency of decision-making units. There are factors among banking indices (such as receivables from resource allocation) that are inevitably created along with desirable outputs. Therefore, the existence of a model for evaluating bank performance in the presence of poor outcomes is very important. In this study, a collection of Iranian commercial bank branches were investigated from this perspective, and radial and non-radial models were provided to measure relative efficiency in the presence of undesirable factors.

**Keywords:** Data Envelopment Analysis, Relative Performance, Undesirable Output.

## 1 Introduction

Mathematics and its related sciences provide accurate and provable methods for understanding the world and complex relations governing its varying components. Operational research is a branch of mathematics that models industrial and social structures, considers their throughput, and provides solutions for the improvement of methods. Data envelopment analysis is a branch of operational research which is used in different fields from industries to hospitals and banks. Data envelopment analysis is a powerful management tool capable of helping management in achieving the prime goal of the organization, i.e. the optimal use and allocation of resources for greater profitability. [1]

Different models have been provided by previous research to measure relative performance. These models are divided into two categories, namely radial and non-radial. In radial models, inputs are equally contracted along the radius of inputs (nature of the input), and outputs are equally expanded along the radius of outputs (nature of the output). In non-radial models, the contraction of all inputs or expansion of all outputs is not equally proportional; in general, the distance of evaluated units from the efficiency border is measured based on different metrics [1,2].

Today, accountability of authorities for their entrusted tasks has been signified. This process is carried out through providing reports on the performance, evaluating it, and/or

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operational audit. Therefore, operational audit and/or performance evaluation, as integral components of accountability, are very important. Banks, as well as financial and credit institutions are among such firms with significant role in the optimal allocation of economic resources in the society. Banks perform their activities through subsidiaries based in different regions. The optimal use of banks' resources, as well as their efficient operation requires that they undertake performance evaluation. In this regard, the performance of all banks' branches should be assessed. According to what has been said about the capability of data envelopment analysis, investigation into the performance of organizations with different branches (such as banks) has been given special attention. The executive responsibilities are upon the branches and the central bank plays the role of supervisor and regulator. Bank performance evaluation is done for some reasons including: the determination of (1) weaknesses and strengths of each branch and creating an appropriate context for a healthy competition between branches and (2) appropriate criteria for budget and resource allocation [3].

This article has been organized as follows. In the second section, a review of radial and non-radial models of data envelopment analysis is provided. The radial and non-radial models of data envelopment analysis in the presence of undesirable data are addressed in the third section. The fourth section deals with the results from the evaluation of 30 branches of a commercial bank in a region in Tehran. Finally, a conclusion is drawn.

## 2 Radial and Non-Radial Models in Efficiency Measurement.

Assume n congruent decision-making units, in which the unit $j^{th}$ uses the input vector $X_j = (x_{1j}, \ldots, x_{mj})$ to produce the output vector $Y_j = (y_{1j}, \ldots, y_{sj})$ where,

$$Y_j \neq 0 \ , Y_j \geq 0 \ , \qquad X_j \neq 0 \ , X_j \geq 0$$

The CCR, developed by Charnes, Cooper and Rhodes is an early data envelopment analysis model that radically measures efficiency [4,5]:

$$\begin{aligned} &Min \quad Z = \theta \\ &s.t. \\ &\sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta x_{ip}, \qquad i = 1, \ldots, m, \qquad (1) \\ &\sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{rp}, \qquad r = 1, \ldots, s, \\ &\lambda_j \geq 0, \qquad j = 1, \ldots, n, \end{aligned}$$

The radial model (1) is an input model used to measure the efficiency of $DMU_p$. In radial models, the efficiency of an evaluated unit is determined by solving two models. To eliminate this weakness, a non-radial model such as SBM is provided as follows to measure the relative efficiency of $DMU_p$ [6].

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$$Min \quad Z = \frac{1-(1/m)\sum_{i=1}^{m} s_i^- / x_{ip}}{1+(1/s)\sum_{r=1}^{s} s_r^+ / y_{rp}}$$

*s.t.*

$$\sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = x_{ip}, \qquad i = 1,...,m, \tag{2}$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = y_{rp}, \qquad r = 1,...,s,$$

$$\lambda_j \geq 0, \qquad j = 1,...,n,$$

By solving this model and/or linear programming of its equivalent, the relative efficiency of decision-making units (DMUs) can be measured. In general, standard models of data envelopment analysis divide DMUs into efficient and inefficient groups. If the optimal value of these models is 1, the evaluated unit will be regarded as efficient. It is worth noting that in the radial (1) and non-radian (2) models, all input and outputs are desirable. It means that they are designed to decrease the number of inputs and increase the number of outputs.

## 3 Practical Example

In this study, 30 branches of an Iranian commercial bank in a region of Tehran were evaluated. Data were extracted in 2014. The desired inputs and outputs are presented in Table 1. Since performance evaluation of the bank's branches is done with different models and a collection of variables and indices, the selection of indices is very important in this type of evaluation.

**Table1** Inputs and outputs of the bank's branches

| Inputs | Outputs |
|---|---|
| Personnel Score<br>Interest Payable | Resources<br>Interest Receivable<br>Facilities<br>Fee Receivable<br>Receivables |

According to Table 1, these 30 congruent branches have five outputs and two inputs. Note that among the outputs, the receivables are those created from facilities granted to clients by bank. The lower amounts of receivables are better. This type of output is called undesirable or bad outcome [7, 8]. In other word, undesirable outputs are those that should be decreased. Therefore, the radial (2) and non-radial (3) models are not useful in the presence of undesirable data.

According to this case study, the following appropriately modified models have been provided for the quantification of relative performance in the presence of undesirable outputs. The radial model in the presence of undesirable outputs, based on CCR nature of the input, is as follows:

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$$
\begin{aligned}
&Min \quad Z = \theta \\
&s.t. \\
&\sum_{j=1}^{n} \lambda_j x_{ij} \le \theta x_{ip}, \qquad i = 1,...,m, \\
&\sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{rp}, \qquad r \in D, \\
&\sum_{j=1}^{n} \lambda_j y_{rj} \le y_{rp}, \qquad r \in UD, \\
&\lambda_j \ge 0, \qquad j = 1,...,n,
\end{aligned}
\tag{3}
$$

The non-radial model in the presence of undesirable outputs, based on SBM, is as follows:

$$
\begin{aligned}
&Min \quad Z = 1 - (1/m)\sum_{i=1}^{m} s_i^- / x_{ip} \\
&s.t. \\
&\sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = x_{ip}, \qquad i = 1,...,m, \\
&\sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = y_{rp}, \qquad r \in D, \\
&\sum_{j=1}^{n} \lambda_j y_{rj} + s_r^+ = y_{rp}, \qquad r \in UD, \\
&\lambda_j \ge 0, \qquad j = 1,...,n,
\end{aligned}
\tag{4}
$$

Where, D is the index set of desirable outputs and UD is the index set of undesirable outputs. In the radial (3) and non-radial (4) models, undesirable outputs have been treated as inputs. Note that in both models, the reduction of undesirable outputs is done through third division modalities.

## 4 Results

The quantified efficiency obtained from the solution of radial model (3) for these 30 branches, using GAMS, is presented in Table 2.

**Table 2** Efficiency of bank branches measured by radial model

| Efficiency | Branch | Efficiency | Branch |
|---|---|---|---|
| 0.809 | Unit 16 | 1.000 | Unit 1 |
| 0.369 | Unit 17 | 0.596 | Unit 2 |
| 0.913 | Unit 18 | 0.613 | Unit 3 |
| 0.896 | Unit 19 | 0.576 | Unit 4 |
| 1.000 | Unit 20 | 0.721 | Unit 5 |
| 1.000 | Unit 21 | 0.526 | Unit 6 |
| 0.702 | Unit 22 | 0.604 | Unit 7 |
| 0.541 | Unit 23 | 0.572 | Unit 8 |
| 0.459 | Unit 24 | 0.884 | Unit9 |
| 1.000 | Unit 25 | 0.919 | Unit 10 |

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| Efficiency | Branch | Efficiency | Branch |
|---|---|---|---|
| 1.000 | Unit 26 | 0.456 | Unit 11 |
| 1.000 | Unit 27 | 0.498 | Unit 12 |
| 1.000 | Unit 28 | 0.681 | Unit 13 |
| 0.655 | Unit 29 | 0.560 | Unit 14 |
| 1.000 | Unit 30 | 0.612 | Unit 15 |

According to Table 2, 8 branches are efficient and 22 branches are inefficient. The efficient branches include 1, 20, 21, 25, 26, 27, 28, and 30, and the remaining is inefficient. Among all inefficient units, branches 10 and 17 are of the highest and lowest degree of inefficiency, respectively. The mean efficiency value of the 30 branches was 0.739.

The quantified efficiency obtained from the solution of non-radial model (4) for these branches, using GAMS, is presented in Table 3.

**Table 3** Efficiency of the bank's branches quantified by non-radial model

| Efficiency | Branch | Efficiency | Branch |
|---|---|---|---|
| 0.749 | Unit 16 | 1.000 | Unit 1 |
| 0.328 | Unit 17 | 0.587 | Unit 2 |
| 0.839 | Unit 18 | 0.546 | Unit 3 |
| 0.822 | Unit 19 | 0.571 | Unit 4 |
| 1.000 | Unit 20 | 0.588 | Unit 5 |
| 1.000 | Unit 21 | 0.474 | Unit 6 |
| 0.698 | Unit 22 | 0.566 | Unit 7 |
| 0.530 | Unit 23 | 0.551 | Unit 8 |
| 0.447 | Unit 24 | 0.760 | Unit9 |
| 1.000 | Unit 25 | 0.855 | Unit 10 |
| 1.000 | Unit 26 | 0.430 | Unit 11 |
| 1.000 | Unit 27 | 0.402 | Unit 12 |
| 1.000 | Unit 28 | 0.632 | Unit 13 |
| 0.613 | Unit 29 | 0.476 | Unit 14 |
| 1.000 | Unit 30 | 0.446 | Unit 15 |

According to Table 3, the number of efficient and inefficient units is exactly the same of the result of Model 3, presented in Table 4. In addition, the efficient units in Model 3 have also been efficient in an evaluation by Model 4, and the amount of efficiency has decreased only in some inefficient units. The mean efficiency of the branches was obtained as 0.697, using a non-radial model. Comparison of radial (1) and non-radial (2) models is presented in figure 1.

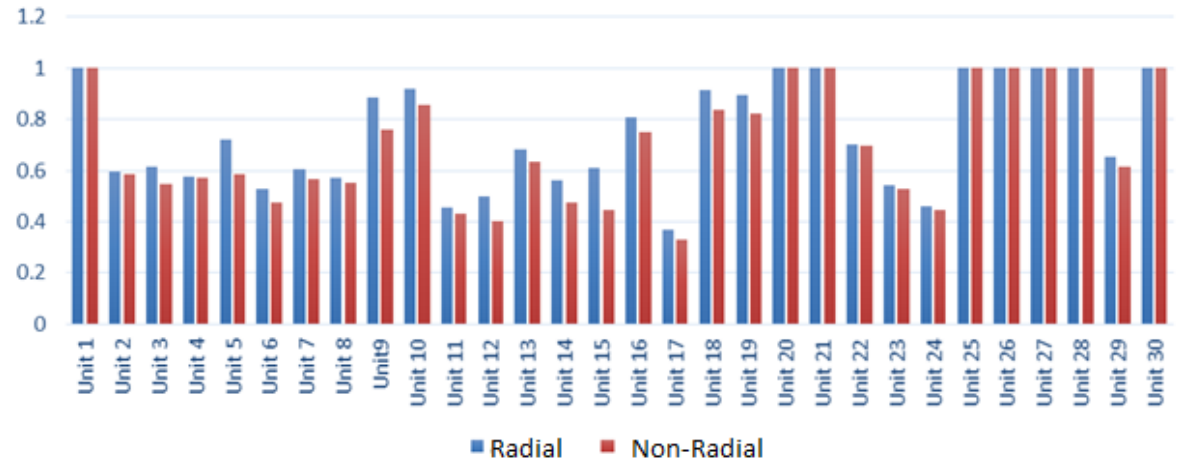



**Fig. 1** Comparison of efficiency of radial and non-radial models

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As seen, the first columns that belong to the radial model (3) are larger or equal to the second columns that belong to non-radial model (4).

## 5 Conclusion

In this study, the efficiency of units was analyzed and measured through a non-radial model. In addition, the existence of undesirable outputs along with desirable outputs was considered in efficiency evaluation and measurement. According to the results from radial and non-radial models, 8 units were identified as efficient and the remaining as inefficient. As was expected, the mean value from the radial model was greater than that from the non-radial one. The results include: (1) the creation of a healthy competitive environment among the bank's branches, and (2) the causes of inefficiency and mechanism of reaching a desirable level of efficiency. Given the inputs and outputs that affects the efficiency, and according to the evaluation data, inefficient branches can be converted into efficient units.

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