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An Economic Production Quantity Model for Defective Items with Sales Return Service, Scrap Items and Rework

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Abstract In this model, we establish an inventory model to determine the optimal inventory replenishment scheme for the economic production quantity (EPQ) model for imperfect, deteriorating items with sales return service under multiple production and rework setup. In one cycle, production process can produces the products in *m* production setups and reworks the defective items in one rework setup. The common assumptions in this model are that all units produced are not perfect and shortages are not allowed. The defective/scrap items are produced during the *m* production setups. The defective items are of two types which are recoverable items and irrecoverable items. The recoverable items are converted into good quality items in rework process and irrecoverable items are considered as scrap (disposable) items. A portion of defective items produced are not successfully screened out internally during the *m* production setups and passed on to customer, thereby causing defect sales returns and reverse logistic from customers back to the manufacturer. The proposed model is demonstrated numerically and the sensitivity analysis is also carried out to study the behavior of the inventory model.

Keywords: Deteriorating Items, Rework, Multiple Production Setups, Sales Return Service, Scrap Items.

1 Introduction

In the global competitive market, it is necessary to produce producing high quality products and attract customers by providing good service. In reality, production processes are often imperfect. For economic and environmental reasons, imperfect quality items are reworked to become serviceable items again. Due to unsuitable inventory condition or other reasons, the remaining good quality items, stored in an inventory, are deteriorating. In order to provide good service to customers, inspection is carried out to screen out imperfect items. However, such inspection may not be perfect and only part of imperfect items can be screened out. The remaining perfect items will then be sold to customers. All the imperfect items are reworked as good quality items and sold it to customers under the consideration that all the imperfect items can be remanufactured as good quality items by rework. Here both perfect quality items and imperfect quality items are considered as deteriorating items. These assumptions will

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underestimate the actual required quantity. Hence, the defective items cannot be ignored in the production process. A portion of defective items produced are not successfully screened out internally during the production process and passed on to customers, thereby causing defect sales returns and reverse logistics from customers back to the manufacturer. One common source of inspection error is from human factors [1,2]. For instant, Anna University of Tamilnadu manufactures answer booklets and sends them to various colleges affiliated to Anna University. The colleges send the booklets to the students through hall supervisor during examination. The student or the supervisor check the booklets and find damages, page number missing, stitching thread is missing, it is not stapled, the page numbers are not in order, the serial number of the answer book is not printed at the top of the title page, etc. The colleges send back those booklets to the university. The university converts the booklets as proper booklets by rework. Therefore, rework process is necessary to convert those defectives into finished goods. The primary operation strategies and goals of most manufacturing firms are to seek high satisfaction to customer's demands and to become a low-cost producer. To reach these goals, the company should be able to effectively utilize resources and minimize costs. Rework is common in semiconductor, pharmaceutical, chemical and food industries. The products are considered as deteriorating items because their utility is lost with time of storage due to price reduction, product useful life expiration, decay and spoilage. In our lot sizing model for deteriorated items with rework, both perfect and imperfect items are deteriorating with time. The production process with rework setups is shown in Fig-1. In this system, items are inspected after production. Good quality items are stocked and sold to customer immediately. Defective items are scheduled for rework. All recoverable items after rework are considered ''as new''. Rework process is not done immediately after the production process, but it waits until a determined number of production setups are over. So deterioration of imperfect items is increased.

The remainder of this paper is organized as follows. In section 2, we give a literature review. In section 3, assumptions and notations are given. The mathematical formulation for this model is given in section 4. Numerical example and sensitivity analysis are given in section 5 and conclusion is drawn in section 6.

2 Literature review

Economic Production Quantity (EPQ) model is one of the prominent research topics in production, inventory control and management. By using EPQ model, optimal quantity of items and optimal production time can be obtained. Classical EPQ model was developed under various assumptions. Thereafter, researchers have extended the model by relaxing one or more of its assumptions. It was assumed that the items produced are of perfect quality items in the classical model. However, imperfect quality items may be produced in reality. Wee et al. [3] extended the model by considering random defective rate. Jaber et al. [4] assumed that the percentage defective per lot reduces according to a learning curve. Mukhopadhyay and Goswami [5] investigated an economic production quantity model for three types of imperfect items with rework. Rezaei and Davoodi [6] considered a supply chain with multiple products and multiple suppliers. Chung et al. [7] proposed an inventory model with two warehouses where one of them was rented. Yassine et al. [8] considered disaggregating the shipments of imperfect quality items in a single production run and aggregating the shipments of imperfect items over multiple production runs. Kumar et al. [9] presented Economic Production Lot Size (EPLS) model with stochastic demand and shortage

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partial backlogging rate under imperfect quality items wherein stochastic imperfect production was assumed. Singh et al. [10] presented a mathematical production inventory model for deteriorating items with time dependent demand rate under the effect of inflation and shortages. Rezaei and Salimi [11] discussed an economic production quantity and purchasing price for items with imperfect quality when inspection shifts from buyer to supplier. An inventory model is developed by Hsu and Hsu [12] to derive an optimal production lot size and backorder quantity for a producer under an imperfect manufacturing process and also they characterized the imperfect manufacturing process by the fraction of defective items at the time of production process. Felix et al. [13] presented a modified EPQ model with deteriorating production system and deteriorating product where rework process was considered at the end of production setup. Mishra et al. [14] considered an inventory model for deteriorating items with time-dependent demand and time varying holding cost under partial backlogging. Jawla and Singh [15] established a multi-item inventory model to derive the optimal inventory replenishment strategy for EPQ model for imperfect, deteriorating items under multi- production setups and one rework setup. They used preservation technology investment system to reduce the deterioration of products. Pal et al. [16] proposed a production inventory model for deteriorating item with ramp type demand allowing inflation and shortages under fuzziness wherein multi-production setup was considered without rework. Jaggi et al. [17] introduced the effect of deterioration on two-warehouse inventory model with imperfect quality items. An incorporated multi-phase supply chain with time-varying demand over a finite planning horizon is studied in Zhao et al. [18] and an algorithm is also given to invent the optimal production inventory policy that minimizes the total inventory cost.

Rework process is also one important issue in reverse logistics where used products are reworked to reduce total inventory cost, wastage and environmental pollution. The earliest research that focused on rework and remanufacturing process was done by Schrady [19]. Since then, researches on rework have attracted many researchers. Khouja [20] considered direct rework for economic lot sizing and delivery scheduling problem (ELDSP). Koh et al. [21] discussed on production inventory models where supplier can fill the demand in two alternatives: either orders new products externally or recovers defective items and rework in the same cycle; and in the second policy, rework is completed after N cycles. Inderfuth et al. [22] considered an EPQ model with rework and deteriorating recoverable products. Yoo et al. [23] developed an EPQ model with imperfect production, imperfect inspection and rework. Widyadana and Wee [24] proposed an EPQ model for deteriorating items with rework which was performed after m production setups. Tai [25] proposed an EPQ model for deteriorating/imperfect product with rework which was performed after a production setup. Sarkar et al. [26] assumed rework for single stage production system. Singh et al. [27] proposed an economic production model for time dependent demand with rework and multiple production setups where production is demand dependent.

We notice that not many studies considered a model with multi-production setups, defective items, rework and sales return service. In this paper, we intend at providing analytic results to solve the said above issues.

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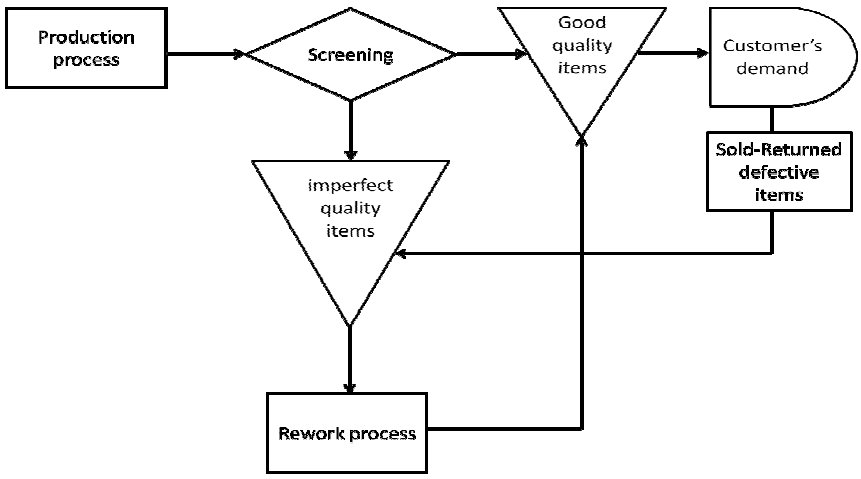

Fig. 1 Production process with rework setups and sales return service.

3 Assumptions and Notations

3.1 Assumptions

1. A single type of product in *m* production setups is considered.
2. Production rate is constant and greater than demand.
3. Proportion of defective items is constant.
4. Defective items, produced during production period and received from customers, are reworked at the end of determined production setup.
5. Proportion of scrap items is less than the proportion of defective items.
6. Screening cost is ignored because it is negligible when compared with other costs.
7. Rework and deterioration rate are constants.
8. There is a replacement for deteriorated items.
9. Shortages and stock outs are not allowed.
10. No machine breakdown occurs in the production run and rework period.
11. All demands are satisfied.
12. Setup time for rework process is zero.

3.2 Notations

$D(t)$ Demand rate (unit/year)

$P(t)$ Production rate (unit/year)

P_r Rework process rate (unit/year)

$\theta(t)$ Deterioration rate (unit/year)

α Percentage of good quality items

β Percentage of recoverable items during rework production period.

x Percentage of sold-returned items during production period.

y Percentage of sold-returned items during non-production period.

m Number of production setup in one cycle

D_i Total deteriorating units (unit)

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K_s Production setup cost ($/setup)
K_r Rework setup cost ($/setup)
h_s Perfect quality items holding cost ($/unit/year)
h_r Imperfect quality items holding cost ($/unit/year)
D_c Deteriorating cost ($/unit)
I_1 Inventory level of perfect quality items in a production period
I_2 Inventory level of perfect quality items in a non -production period
I_{r1} Inventory level of imperfect quality items in a production period
I_{r2} Inventory level of imperfect quality items in a non - production period
I_{r3} Inventory level of imperfect quality items in a rework production period
I_{t1} Total Inventory level of perfect quality items in a production period
I_{t2} Total Inventory level of perfect quality items in a non - production period
I_{t3} Total Inventory level of perfect quality items in a rework production period
I_{t4} Total Inventory level of perfect quality items in a rework non-production period
TTI_1 Total Inventory level of imperfect quality items in a production period
I_{v1} Total Inventory level of imperfect quality items in m production periods
TTI_2 Total Inventory level of imperfect quality items in a non-production period
I_{v2} Total Inventory level of imperfect quality items in m non - production period
I_{v3} Total Inventory level of imperfect quality items in a rework setup production period
TRI Total Inventory level of imperfect quality items
I_{Mr} Maximum Inventory level of imperfect quality items in production setups
I_{Er} Maximum inventory level of imperfect quality items when rework process started
T_1 Regular production period
T_2 Non - production period
T_3 Rework process period
T_4 Non rework process period
TCT Total cost per unit time
C_r Cost of rejection per unit

4 Formulation of the inventory model

The Inventory level of perfect quality items in three production setups and one rework setup is shown in Fig-2. The cycle begins with zero inventory and starts at time t = 0. Production is performed during T_1 time period. Since the production quantity is not perfect, a percentage αP imperfect items is assumed to occur during the regular production process(T_1). The amount of imperfect quality items produced per unit time is$(1-\alpha)P$. Since the screening work is not perfect, a percentage βD imperfect items is assumed to occur

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during the non-production process (T_2). The amount of imperfect items received from the customer per unit time during non-production period is $(1-\beta)D$. The rework process starts after m-production setups. The rework process is performed in T_3 time period. Since production processes of material and reproduce of imperfect items are different, rework rate is not the same as the production rate.

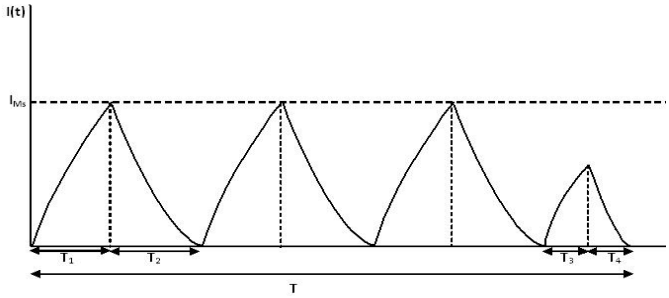

Fig. 2 Inventory level of perfect quality items in 3 production setups and 1 rework setup.

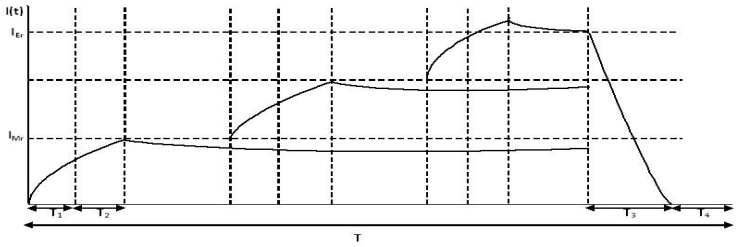

Fig. 3 Inventory level of imperfect items in 3 production setups and 1 rework setup.

The Inventory level of perfect quality items in a production period can be formulated as:

$$\frac{dI_1(t_1)}{dt_1} + \theta I_1(t_{1)} = \alpha P - D \qquad 0 \le t_1 \le T_1 \tag{1}$$

Since $I_1(0)=0$, the inventory level of perfect quality items in a production period is

$$\mathrm{I}_1(\mathrm{t}_1) = \frac{\alpha P - D}{\theta}\left[1 - e^{-\theta t_1}\right] \qquad 0 \le t_1 \le T_1 \tag{2}$$

The total inventory in a production up time can be modeled as

$$\mathrm{I}_{t1}(\mathrm{t}_1) = \frac{\alpha P - D}{\theta}\int_0^{T_1}\left[1 - e^{-\theta t_1}\right] dt_1 \tag{3}$$

For small value of θT_1 and using Taylor series approximation, we get

$$\mathrm{I}_{t1} = \frac{(\alpha P - D)T_1^2}{2}\,. \tag{4}$$

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The inventory level of perfect quality items in a non-production period is represented by

$$\frac{dI_2(t_2)}{dt_2} + \theta I_2(t_{2)} = -D \qquad 0 \le t_2 \le T_2 \tag{5}$$

Since $I_2(T_2) = 0$ and using similar procedure we get the total inventory in a non-production period can be represented as

$$I_{t2}(t_2) = \frac{D}{\theta}\left[e^{\theta(T_2 - t_2)} - 1\right] \tag{6}$$

$$I_{t2} = \frac{DT_2^2}{2}. \tag{7}$$

Since $I_1 = I_2$ when $t_1 = T_1$ and $t_2 = 0$, we get

$$\frac{\alpha P - D}{\theta}\left[1 - e^{-\theta T_1}\right] = \frac{D}{\theta}\left[e^{\theta T_2} - 1\right]$$

$$T_2 \cong \frac{(\alpha P - D)\left[2T_1 - \theta T_1^2\right]}{2D} \tag{8}$$

The inventory level of perfect quality items during rework production period is represented by

$$\frac{dI_3(t_{3)}}{dt_3} + \theta I_3(t_{3)} = \gamma P_r - D \qquad 0 \le t_3 \le T_3 \tag{9}$$

$$I_3(t_{3)} = \frac{\gamma P_r - D}{\theta}\left[1 - e^{-\theta t_3}\right] \qquad 0 \le t_3 \le T_3 \tag{10}$$

The total inventory of perfect quality items in a rework production up time period is calculated as

$$I_{t3} = \frac{(\gamma P_r - D)\, T_3^2}{2}. \tag{11}$$

The inventory level of perfect quality items during rework non-production period is

$$\frac{dI_4(t_4)}{dt_4} + \theta I_4(t_4) = -D \qquad 0 \le t_4 \le T_4 \tag{12}$$

$$I_4(t_{4)} = \frac{D}{\theta}\left[e^{\theta(T_4 - t_4)} - 1\right] \qquad 0 \le t_4 \le T_4 \tag{13}$$

The total inventory perfect quality items in a rework non-production period is

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$$\mathrm{I}_{t4} = \frac{D\mathrm{T}_4^{\,2}}{2}. \tag{14}$$

Since $\mathrm{I}_3 = \mathrm{I}_4$ when $t_3 = T_3$ and $t_4 = 0$, we get

$$\frac{\gamma P_r - D}{\theta}\left[1 - e^{-\theta T_3}\right] = \frac{D}{\theta}\left[e^{\theta T_4} - 1\right]$$

$$T_4 \cong \frac{(\gamma P_r - D)\left(2T_3 - \theta T_3^2\right)}{2D}. \tag{15}$$

The inventory level of imperfect quality items is shown in Fig-3. The inventory level of imperfect quality items in a production period can be modeled as

$$\frac{dI_{r1}(t_{r1})}{dt_{r1}} + \theta I_{r1}(t_{r1)} = (1-\alpha)P + xD \qquad 0 \le t_{r1} \le T_1 \tag{16}$$

Since $\mathrm{I}_{\mathrm{r1}}(0) = 0$, the inventory level of imperfect quality items in a production period is

$$I_{r1}(t_{r1)} = \frac{(1-\alpha)P + xD}{\theta}\left(1 - e^{-\theta t_{r1}}\right) \qquad 0 \le t_{r1} \le T_1 \tag{17}$$

Using Taylor series approximation, the total inventory level of imperfect quality items in a production up time in one setup is

$$\mathrm{TTI}_1 = \frac{[(1-\alpha)P + xD]T_1^2}{2}. \tag{18}$$

Since there are m production setups in one cycle, the total inventory level of imperfect quality items in one cycle is:

$$\mathrm{I}_{\mathrm{v1}} = \frac{m\left[(1-\alpha)P + xD\right]T_1^2}{2}. \tag{19}$$

The inventory level of imperfect quality items in a non-production period is

$$\frac{dI_{r2}(t_{r2})}{dt_{r2}} + \theta I_{r2}(t_{r2}) = yD \qquad 0 \le t_{r2} \le T_2 \tag{20}$$

$$I_{r2}(t_{r2}) = \frac{y}{D}\left(1 - e^{-\theta t_{r2}}\right) \qquad 0 \le t_{r2} \le T_2 \tag{21}$$

Total inventory level of imperfect quality items during non-production period is

$$TTI_2 = \frac{yD}{2}T_2^{\,2} \tag{22}$$

Since there are m production setups in one cycle, the total inventory of imperfect quality items in one cycle is:

$$I_{v2} = \frac{myD}{2}T_2^{\,2} \tag{23}$$

The initial inventory level of imperfect quality items in each production setup is equal to I_{Mr} and it can be modeled as:

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$$\mathrm{I_{Mr}} = \mathrm{I_{r1}}\left(\mathrm{T_1}\right)+\mathrm{I_{r2}}\left(\mathrm{T_2}\right)= \frac{\left[\left(1-\alpha\right)\mathrm{P}+xD\right]\left\{2\mathrm{T_1}-\theta T_1^2\right\}}{2}+\frac{yD\left[2\mathrm{T_2}-\theta T_2^{\,2}\right]}{2}. \tag{24}$$

The waited inventory level of imperfect quality items is

$$\frac{dI_{r3}\left(t_{r3}\right)}{dt_{r3}}+\theta I_{r3}\left(t_{r3}\right)=0 \qquad 0\le t_{r3}\le\left(m-1\right)T_1+\left(m-1\right)T_2 \tag{25}$$

Since the inventory level of imperfect quality items when $t_{r3}=0$ is equal to $\mathrm{I_{Mr}}$, then the waited inventory level of imperfect quality items is

$$I_{r3}\left(t_{r3}\right)=\mathrm{I_{Mr}}\,e^{-t_{r3}} \tag{26}$$

The total waited inventory level of imperfect quality items in $(m-1)$ non-production period is

$$I_{v3}=\sum_{k=1}^{m}\int_{0}^{(k-1)(T_1+T_2)}\mathrm{I_{Mr}}\;\mathrm{e}^{-\theta t_{r3}}\;\mathrm{d}t_{r3} \tag{27}$$

$$I_{v3}=\sum_{k=1}^{m}\mathrm{I_{Mr}}\left\{\left(\mathrm{k}-1\right)\left(T_1+T_2\right)-\frac{\theta\left(k-1\right)^2\left(T_1+T_2\right)^2}{2}\right\} \tag{28}$$

Inventory level of imperfect quality items in the end of production cycle is equal to maximum inventory level of imperfect quality items in a production setup reduced by deteriorating rate during production up time and down time. The inventory level of imperfect quality items can be formulated as follows:

$$\mathrm{I_{Er}}=\sum_{k=1}^{m}\mathrm{I_{Mr}}\;\mathrm{e}^{-\theta\left[\left(\mathrm{k}-1\right)\left(\mathrm{T_1}+\mathrm{T_2}\right)\right]} \tag{29}$$

Using Taylor series approximation, we get

$$\mathrm{I_{Er}}=\sum_{k=1}^{m}\left\{\left[\frac{\left[\left(1-\alpha\right)\mathrm{P}+xD\right]\left\{2\mathrm{T_1}-\theta T_1^2\right\}}{2}+\frac{yD\left[2\mathrm{T_2}-\theta T_2^{\,2}\right]}{2}\right]\times\left[1-\theta\left(\mathrm{k}-1\right)\left(\mathrm{T_1}+\mathrm{T_2}\right)+\frac{\theta^2\left(\mathrm{k}-1\right)^2\left(\mathrm{T_1}+\mathrm{T_2}\right)^2}{2}\right]\right\} \tag{30}$$

The inventory level of imperfect quality items in a rework period can be represented as:

$$\frac{dI_{r4}(t_{r4})}{dt_{r4}}+\theta I_{r4}(t_{r4})=-P_r \qquad 0\le t_{r4}\le T_3 \tag{31}$$

$$I_{r4}(t_{r4})=\frac{P_r}{\theta}\left[e^{\theta(T_3-t_{r4})}-1\right] \quad 0\le t_{r4}\le T_3 \tag{32}$$

The total inventory level of imperfect quality items in a rework period can be modeled as:

$$I_{v4}\left(t_{r4}\right)=\int_{t_{r4}=0}^{T_3}\frac{P_r}{\theta}\left[e^{\theta(T_3-t_{r4})}-1\right]dt_{r4}$$

$$I_{v4}=\frac{P_r}{2}T_3^2 \tag{33}$$

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When $t_{r4} = 0$, the number of imperfect quality items inventory is equal to $\mathrm{I_{Er}}$.
From equation (32),

$$\mathrm{I_{Er}} = \frac{P_r}{\theta}\left[e^{\theta T_3} - 1\right] \tag{34}$$

Using Taylor series approximation and $\frac{(\theta T)^2}{2} \ll 1$, we get

$$T_3 = \frac{\mathrm{I_{Er}}}{P_r} \tag{35}$$

$$T_3 = \frac{1}{P_r}\sum_{k=1}^{m}\left\{\left[\frac{\left[(1-\alpha)\mathrm{P} + xD\right]\left\{2\mathrm{T_1} - \theta T_1^2\right\}}{2} + \frac{yD\left[2\mathrm{T_2} - \theta T_2^2\right]}{2}\right]\times \left[1 - \theta(\mathrm{k}-1)(\mathrm{T_1} + \mathrm{T_2}) + \frac{\theta^2(\mathrm{k}-1)^2(\mathrm{T_1} + \mathrm{T_2})^2}{2}\right]\right\} \tag{36}$$

The total imperfect quality items in one cycle can be formulated as:

$$TRI = I_{v1} + I_{v2} + I_{v3} + I_{v4}$$

$$TRI = \left[\frac{m\left[(1-\alpha)P + xD\right]T_1^2}{2} + \frac{myD}{2}T_2^2 + \frac{P_r}{2}T_3^2 + \sum_{k=1}^{m}\left\{(\mathrm{k}-1)(T_1+T_2) - \frac{\theta(k-1)^2(T_1+T_2)^2}{2}\right\}\left\{\frac{\left[(1-\alpha)\mathrm{P} + xD\right]\left\{2\mathrm{T_1} - \theta T_1^2\right\}}{2} + \frac{yD\left[2\mathrm{T_2} - \theta T_2^2\right]}{2}\right\}\right] \tag{37}$$

The number of deteriorating item = the number of total items produced – (the number of total demands + scrap items)
Total deteriorating units can be modeled as:

$$D_i = \left(m\alpha PT_1 + \gamma P_r T_3\right) - \left[D\left(m\left(T_1+T_2\right) + T_3 + T_4\right) + \left(1-\gamma\right)P_r\right] \tag{38}$$

The total inventory cost consists of production setup cost, rework setup cost, good quality items inventory cost, imperfect quality items inventory cost, deteriorating cost and rejection cost. The total inventory cost per unit time can be modeled as follows:

$$TC\left(m, T_1\right) = \frac{mk_s + k_r + h_s\left[m\left(\mathrm{I_{t1}} + \mathrm{I_{t2}}\right) + \mathrm{I_{t3}} + \mathrm{I_{t4}}\right] + h_r\left(TRI\right) + D_c D_i + C_r\left(1-\gamma\right)P_r}{\left[m\left(\mathrm{T_1} + \mathrm{T_2}\right) + \mathrm{T_3} + \mathrm{T_4}\right]} \tag{39}$$

The optimal solution must satisfy the following condition:

$$\frac{\partial\left[TC\left(m, T_1\right)\right]}{\partial T_1} = 0$$

And the optimal solution of m, denoted by m^*, must satisfy the following condition:

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$$TC\left(m^*-1,T_1\right)\geq TC\left(m^*,T_1\right)\leq TC\left(m^*+1,T_1\right)$$

Since the cost function equation (39) is a nonlinear equation and the second derivative of equation (39) with respect to T_1 is complicated, closed form solution of (39) cannot be derived. However, by means of mathematical software, one can indicate that equation (39) is convex for a small value of T_1. The optimal T_1 value can be obtained using Mathematica software.

5 Numerical example and sensitivity analysis

In this section, a numerical example and sensitivity analysis are given to illustrate the model. Let $K_s = \$\,4$ per production setup, $K_r = \$\,3$ per rework setup, $P_r = 18$ units per unit time , $C_r = \$\,1$ per unit, $h_s = \$6$ per unit per unit time, $h_r = \$3$ per unit per unit time, $D_c = \$1$ per unit, $P = 1000$, $D = 983$, $x = 0.001$, $y = 0.01$, $\alpha = 0.8$, $\beta = 0.01$, $\theta = 0.01$. The total cost per unit time for varying T_1 is shown in Fig-4. Fig-4 shows that the total cost per unit time is convex for small values of T_1. The optimal total cost is equal to \$ 995.0119 when $T_1^* = 0.0156$ and $m^* = 9$.

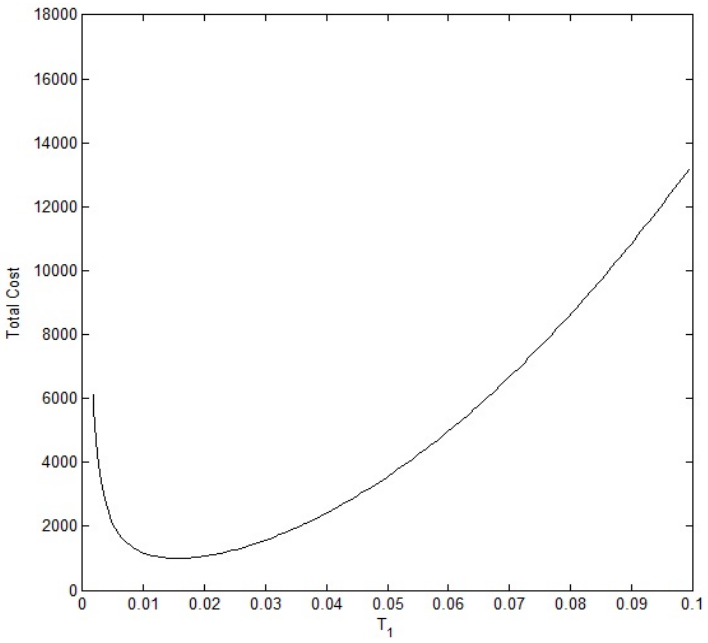

Fig. 4 Total cost per unit time in varies of T_1

The sensitive analysis is performed by changing each of the parameters by -40%, -20%, +20% and +40%. One parameter is taken at a time and the remaining parameters are kept unchanged. The m and T_1 values for different values of parameters are shown in table-1.Table-1 shows that the number of production setup is not sensitive to the changes in parameters except α. The optimal production setup (m^*) is not sensitive to other parameters.

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The optimal production time (T_1^*) increases when changing the parameter k_s and P_r by +20% and +40%, D by +20%, C_r by +40%, h_r by -20% and D_c by +40%. The optimal production time (T_1^*) extremely increases when changing the parameter D by +40%, h_r by -40% and α by +20%. The optimal production time (T_1^*) decrease when changing the parameters k_s, P_r C_r and D_c by -40% and -20% also changing the parameter P, h_r, D_c by +20% and P by + 40%. The optimal production time (T_1^*) moderately decrease when changing the parameters α by -40% and -20%. But optimal production time is insensitive with the parameters x, y, k_r, β, h_s and θ.

The optimal production period for varying parameters is shown in Fig-5. The Fig-5 shows that the optimal production period (T_1^*) is insensitive to changes in x, y, k_r, β, h_s, θ and temperately sensitive to changes in C_r, h_r, P, D and insensitive to changes in the other parameters.

Table 1 Sensitivity analysis of m and T_1.

Parameter	- 40 % changed		- 20 % changed		+ 20 % changed		+ 40 % changed	
	m	T1	m	T1	m	T1	m	T1
Ks	9	0.0144	9	0.0150	9	0.0162	9	0.0162
Kr	9	0.0156	9	0.0156	9	0.0156	9	0.0156
Pr	9	0.0144	9	0.0150	9	0.0162	9	0.0162
P	9	0.0222	9	0.0180	9	0.0138	9	0.0120
D	9	0.0126	9	0.0144	9	0.0168	9	0.0174
α	9	0.0132	9	0.0138	9	0.0246	-	-
x	9	0.0156	9	0.0156	9	0.0156	9	0.0156
y	9	0.0156	9	0.0156	9	0.0156	9	0.0156
β	9	0.0156	9	0.0156	9	0.0156	9	0.0156
C_r	9	0.0150	9	0.0150	9	0.0156	9	0.0162
h_s	9	0.0156	9	0.0156	9	0.0156	9	0.0156
h_r	9	0.0186	9	0.0168	9	0.0144	9	0.0138
D_c	9	0.0150	9	0.0150	9	0.0150	9	0.0168
θ	9	0.0156	9	0.0156	9	0.0156	9	0.0156

The optimal total cost per unit time for varying parameters is shown in table-2. The table-2 shows that the total cost per unit time marginally increases when changing the parameters β, h_s and θ by -40%, -20% and y by -40%. The total cost per unit time increases when changing the parameters P by -20%, k_s, k_r, P_r, C_r, h_r and D_c by +20% and k_r, C_r and D_c by +40%. The total cost per unit time decreases when changing the parameters k_r, C_r, D_c by -40% and k_s, k_r, P_r, C_r, h_r, D_c by -20% and P by +20%. The total cost per unit time marginally decreases when changing the parameters x by -40%, -20% and y , β, h_s , θ by +40%, +20%. The total cost is more sensitive when change the parameters k_s, P_r, h_r by -40%, +40%, D by +20, -20% and α by +20. But the paremeter α is higuhly sensitive when changing the paramaters. The Fig-6 shows that the parameters k_s, P_r, D, C_r, h_r are senditive

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with the total cost while there is a fluxuation when changing the other parameters.

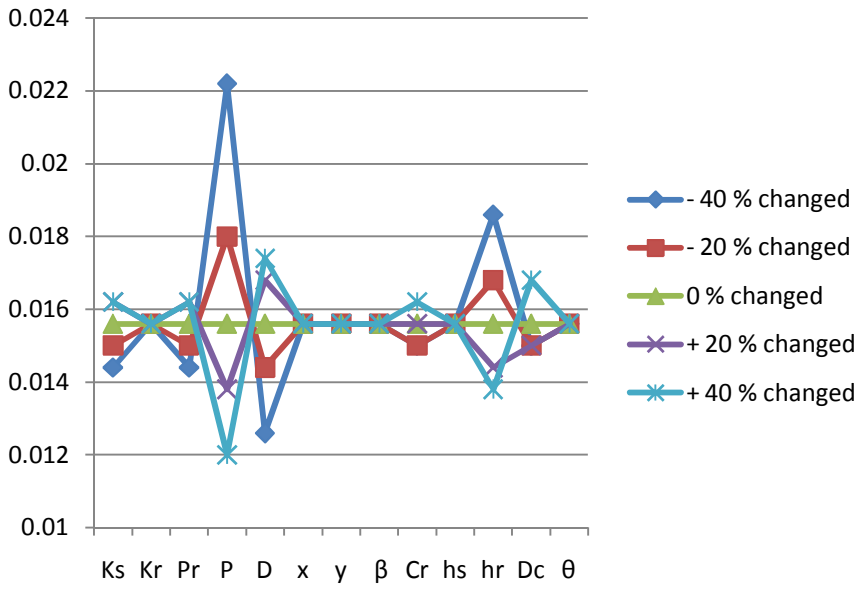

Fig. 5 T_1 sensitivity analysis

Table 2 Sensitivity analysis for the total cost per unit time($)

Parameter	- 40 % changed	- 20 % changed	+ 20 % changed	+ 40 % changed
K_s	863.9886	930.6680	1057.3847	1117.9641
K_r	984.5265	989.7692	1000.2546	1005.4973
P_r	861.5685	930.4676	1057.0295	1117.0047
P	1138.3179	1055.6329	948.2113	910.6222
D	728.0894	868.8067	1110.1099	1216.1186
α	1928.1003	1409.2430	515.6490	-
x	994.3180	994.6650	995.3588	995.7057
y	996.2787	995.6453	994.3785	993.7451
β	996.4358	995.7238	994.3001	993.5883
C_r	931.3224	963.7114	1026.1538	1056.7790
h_s	999.0162	997.0141	993.0097	991.0076
h_r	830.6650	919.6717	1061.1212	1120.1046
D_c	931.8201	963.9602	1025.8948	1056.2416
θ	996.6293	995.8200	994.2051	993.3995

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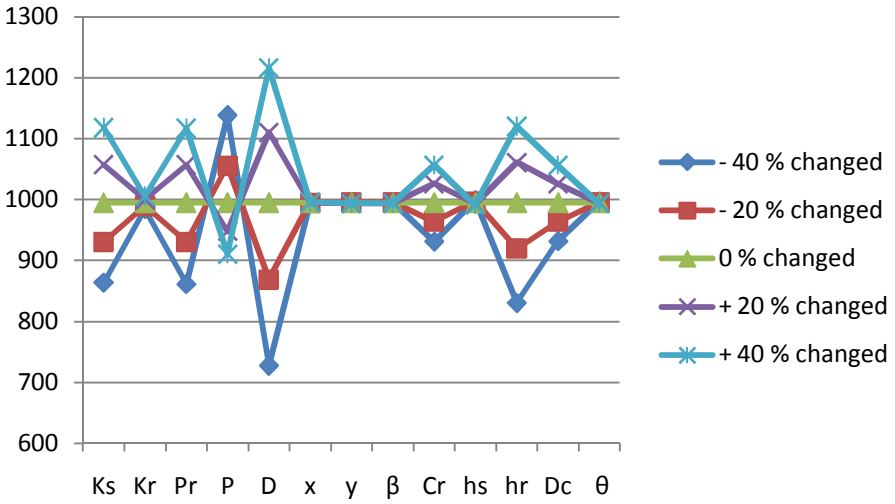

Fig. 6 Total cost per unit time sensitivity analysis

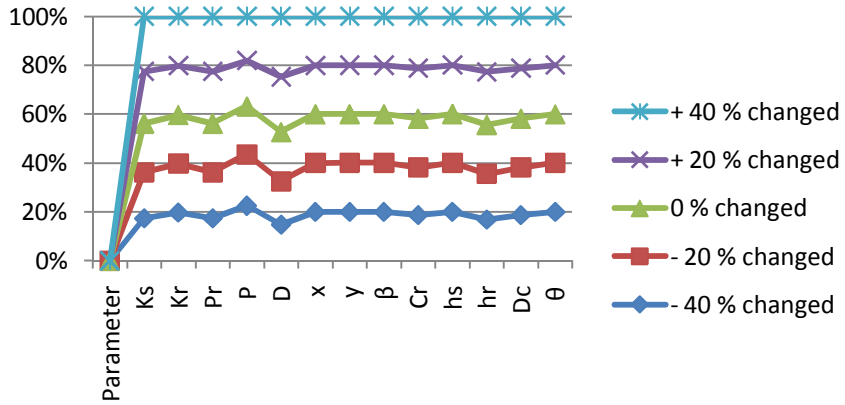

Fig. 7 Total cost against parameters

6 Conclusion

In practices, often both production and inspection processes of a manufacturer are not perfect, thereby producing and passing some imperfect items to customers causing subsequent defect sales returns. Most of the existing imperfect quality inventory models, however, have not dealt with such important practical situations involving both imperfect production and imperfect screening processes. A major reason of reverse logistic and green supply chain is rework which reduces the production cost and environmental problem. Therefore, we present an EPQ model for imperfect quality items with rework and defect sales return service that determines an optimal production setup. The inspection error of falsely not screening out a proportion of defects, thereby passing them on to customers and consequently resulting in customers defect sales returns due to quality dissatisfaction. The proposed model can assist the manufacturer and retailer in accurately determining the optimal production setup, cycle time and total inventory cost. Moreover, the proposed inventory model can be used in inventory control of certain items such as fashionable commodities, stationary stores, paper

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industry, cool drinks company and others. The sensitivity analysis show that the optimal production time is sensitive to changes in the production rate, rework process rate, demand rate and holding cost of imperfect items. The deteriorating cost affects the total cost per unit time; however, it is not significant. The sensitivity also shows that total cost increases when decreasing demand, percentage of perfect items, holding cost of imperfect items and deteriorating rate and the total cost increases when increasing the production setup cost, rework setup cost rework process rate, production rate, rejection cost, holding cost of imperfect items and deteriorating cost.

This approach can also be extended to linearly increasing/decreasing demand, two cases of rework process, two types of inspection error, stock-dependent demand, selling price dependent demand under the effect of preservation technology and learning environment.

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