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A New Method for Ranking Distribution Companies with Several Scenarios Data by Using DEA/MADM

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Abstract In Data Envelopment Analysis, uncertain data are the inseparable part of real models. Natural models usually deal with uncertain and probable data. Many researchers prioritize these kinds of data. For instance, they study fuzzy data, interval data, probabilistic models etc. In this article, we proposed a method in which the decision making units are uncertain in their inputs and outputs. In the proposed method, it is supposed that the inputs and outputs have different scenarios with specific probability occurrence. In this article, applying the VIKOR technique rather than point estimate, the decision making units whose inputs and outputs have different scenarios with specific probability occurrence are ranked. It finds the compromise ranking list and the compromise solution obtained with the given weights from a set of alternatives in the presence of conflicting criteria. In this article, we combine the DEA and the VIKOR method to rank DMUs with different scenarios with a specified probability for input and output data. To illustrate the ability of proposed combined method, a numerical example of 38 Iranian electricity distribution companies is considered.

Keywords: Power Distribution, Data Envelopment Analysis, VIKOR Method, Scenario Base Data.

1 Introduction

Data envelopment analysis (DEA) is a mathematical programming method for evaluating performance and measuring peer decision making units (DMUs). This method discovers the optimal combination of inputs and outputs for independent and peer decision making units (DMUs). Consider n DMUs with nonnegative vectors of $x_j = (x_{1j}, x_{2j}, \dots, x_{sj})$ as an input vector and $y_j = (y_{1j}, y_{2j}, \dots, y_{rj})$, j=1,…,n , as an output vector, In which x_{ij} shows the value of ith input and y_{ij} represents the value of ith output for DMU_j , j= 1,…n . Efficiency score of DMU_d is defined as $e_d(u,v) = y_d u^t / x_d v^t$. This ratio is the weighted sum of its outputs to

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the weighted sum of its inputs. In addition, u and v are the row vectors of input and output weights. Podinovski and Athanassopoulos [1] and Podinovski [2] suggested $f_d(u,v)$ as below:

$$f_d(u,v) = e_d(u,v) / \max\{ e_j(u,v),\ \ u,\ v \geq 0 \mid j = 1,\dots,n\} \tag{1}$$

which is called relative efficiency of DMU_d . It is obvious that f_d^* does not exceed 1 for any DMU_j. The optimal solution for the model (1) originates input and output vectors which generally differ from one DMU to another. This model is the same as the CCR DEA model, introduced by Charnes, Cooper, and Rhodes [3]. A number of different DEA models, after proposing the CCR model, have been introduced by Seiford and Thrall [4], Seiford [5], and Cooper, Seiford and Tone [6]. These models have wide applications in various performance evaluation problems. In all traditional methods, all inputs and outputs are in values which are exact numerically values form while observed values of inputs and outputs data in real problems are usually uncertain, interval or based on different scenarios. In order to study this issue, different approaches have been proposed in the literature. O'Neal, Ozcan, & Yanking [7] offered the exclusion of the DUMs, to have vague data in order to calculate efficiency. This affects efficiency score and relative efficiency of other DMUs. Therefore, it is not acceptable to factoring out DMUs from our calculation [8]. In some approaches, imputation technique is used (e.g. data average for other DMUs). Using this method in DEA may cause misleading results because of stability problems, where a unit accepting a small change may alter its classification status from efficient to inefficient or vice versa. Accordingly, it is not a reliable method for DEA [6].

A Stochastic approach is also applied for uncertain data in DEA. Stochastic programming has undergone many theoretical developments since the 1950s, starting with the pioneering works of Dantzig [9] and Beale [10]. Nevertheless, this approach suffers from its drawbacks. One such drawback with this method is determining probability distribution function in the absence of sufficient empirical evidence [11]. Another approach, which was proposed by Kuosmanen [12], applies dummy variables instead of missing data. The output is considered to be zero and input data is considered to be a relatively large number with regard to the other input numbers and using a weight limit to reduce the impact of that data on the efficiency of other DMUs. Two other approaches to vague or indistinct data are using the fuzzy and interval DEA, developed by Sengupta [11] and Cooper, Park and Yu [13] respectively.

Cooper et al. [13] have developed an interval method by converting the DEA model to the form of linear programming allowing the combination of exact and non-exact data. Assessing upper and lower limits of the relative efficiency of DMUs is one of the difficulties in the interval method Regardless to this difficulty. Despite this difficulty, some researchers have developed numerous interval methods [14-19].

What motivated Sengupta [11] to present a fuzzy method and a fuzzy linear programming as a practical approach in such circumstances were the uncertainties over the stability of efficiency frontier and probabilistic feasibility of inequality constraints in DEA. In a fuzzy approach, several mathematical programming methods including probabilistic planning and α-cut approaches are applied to evaluate the relative efficiency of DMUs. Although there is a sharp growth in the complexity of fuzzy method in some circumstances, many researchers studied on fuzzy DEA [20- 30].

By considering uncertainty in output parameters for the performance assessment of electricity distribution companies, Sadjadi and Omrani [31] have proposed a robust model for DEA considering uncertainty in output parameters for the performance assessment of electricity distribution companies. Shokouhi, Hatami-Marbini, Tavana, and Saati [32] have proposed an approach based on a robust optimization model where the input and output parameters are constrained to be within an uncertainty set under the assumption of a worst case efficiency defined by the uncertainty set and it's supporting constraint. Hafezalkotob et al. [33] proposed RDEA which is based upon the discrete robust optimization approaches proposed by Mulvey et al. [34] that utilizes probable scenarios to capture the effect of ambiguous data in the case study.

Multi-criteria optimization is the method of identifying the best solution which is feasible for alternatives with some criteria representing different effects. Practical problems are often considered by several non-commensurable and different criteria and there may be no solution which can simultaneously satisfy all criteria simultaneously. Thus, the solution is a set of non-inferior solutions, or a compromise solution according to the decision maker's preferences. The compromise solution was established by Yu [35] and Zeleny [36] for a problem with conflicting criteria and it can help the making units to achieve a final solution. The compromise solution is a feasible solution, it means it is the closest possibility to the ideal solution, and compromise in this term means an agreement which has been established by mutual concessions.

A multi attribute decision making (MADM) problem can be defined as below:

Table 1 A multi attribute decision making (MADM) problem

	C_1	C_2	$\cdots$	C_n
A_1	f_{11}	f_{12}	$\cdots$	f_{1n}
A_2	f_{21}	f_{22}	$\cdots$	f_{2n}
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
A_m	f_{m1}	f_{m2}		f_{mn}
	W=[w_1 , w_2, …,w_n]			

while decision making units must choose a possible alternative from A1, A2, . . ., Am, alternative performance is evaluated by criteria C1, C2, . . ., Cn. f_{ij} is the rating of alternative Ai with respect to criterion Cj, wj is the weight of criterion Cj [37-39]. In classical MCDM methods, the ratings and the weights of the criteria are known precisely, whereas in the real world, in an imprecise and uncertain environment, it is an unrealistic assumption that the knowledge and representation of a decision maker or expert are so precise. For example, human judgment including preferences is often vague; consequently, and decision maker (DM) cannot predict their preference with exact numerical values. Conditions like these, specifying the exact value of the attributes is difficult or impossible. Therefore, in order describe and treat imprecise and uncertain elements which are existing in a decision making problem, fuzzy and stochastic approaches are applied frequently.

2 The CCR Model and Efficiency

The model CCR is a model in input nature for n DMUs. Each DMU is specified by some inputs $x_j = (x_{1j}, x_{2j}, \ldots, x_{mj})$ and outputs $y_j = (y_{1j}, y_{2j}, \ldots, y_{sj})$ in which all the inputs and outputs are nonnegative and all the DMUs are independent and peer. Efficiency score of DMU_d is achieved by the model 2 which is known as CCR model:

$$
\begin{aligned}
&\text{Min } \theta_d \\
&\sum_{j=1}^{n} \lambda_j \, x_{ij} \le \theta \, x_{io} \quad i = 1,\dots,m \\
&\sum_{j=1}^{n} \lambda_j \, y_{rj} \ge y_{ro} \quad r = 1,\dots,s \\
&\lambda_j \ge 0 \quad j = 1,\dots,n
\end{aligned}
\tag{2}
$$

$\theta=1$, $\lambda_j = 0$ ($j \neq d$) and $\lambda_d = 1$ is feasible solution for the model 2. Therefore, the optimal solution for this model which is shown by θ^* is smaller or equal to 1. It is clear that if θ_d^* is equal to 1, DMU is efficient and if it is smaller than 1, it is inefficient. The Dual for the model 2 is as follows:

$$
\begin{aligned}
&\text{Max } E_{dd} = \sum_{r=1}^{s} u_{rd} y_{rd} \\
&\sum_{i=1}^{m} v_{id} x_{id} = 1 \\
&\sum_{r=1}^{s} u_{rj} y_{rj} - \sum_{i=1}^{m} v_{ij} x_{ij} \le 0 \quad j = 1,\dots,n \\
&u_{rd} \ge 0,\ v_{id} \ge 0 \quad r = 1,\dots,s \quad i = 1,\dots,m
\end{aligned}
\tag{3}
$$

In this model, E_{dd}^* is equal to θ_d^* in the model CCR for DMU_d.

3 The VIKOR Method

The VIKOR method is a technique to be implemented within MCDM problem. it was introduced as a multi attribute method to solve a discrete decision making problem with different units and conflicting criteria. This method is used for ranking and selecting from several alternatives. The VIKOR method determines compromise solution for a problem with conflicting criteria, which can help the decision makers to find a final decision.

Each alternative is evaluated according to each criterion function; the compromise ranking could be performed by comparing the measure of closeness to the ideal alternative. Consider m alternatives A1, A2, . . ., Am. For alternative A_i, the rating of the jth aspect is denoted by f_{ij}. It means f_{ij} is the value of jth criterion function for the ith alternative ; n is the number of criteria. The VIKOR method is started with the equation (4):

$$
L_{pi} = \{\sum_{j=1}^{n} [(f_j^* - f_{ij}) / \left(f_j^* - f_j^-\right)]^p\}^{1/p} \quad 1 \quad i = 1,\dots,\ m
\tag{4}
$$

In the VIKOR method S_i And formulate ranking measure. The solution obtained by min S_i is with a maximum group utility (‘‘majority” rule), and the solution obtained by min is with R_i a minimum individual regret of the ‘‘opponent”. Steps of the compromise ranking algorithm of the VIKOR method are as follows:

Step1: Find the best f_j and the worst f_j values of all criterion functions j= 1,2,. . .,n. Then, find f_j^* and f_j^- as follows:

$$
f_j^* = \max_j f_{ij} \quad , \quad f_j^- = \min_j f_{ij}
$$

Step2: Compute the values S_i and R_i; i = 1,2,. . .,m,:

$$S_i = \sum_{j=1}^{n} w_j \left(f_j^* - f_{ij}\right) / \left(f_j^* - f_j^-\right) \quad (5)$$

$$R_i = \max_j w_j \left(f_j^* - f_{ij}\right) / \left(f_j^* - f_j^-\right) \quad (6)$$

Where w_j are the weights of criteria, expressing their relative importance.
Step3: Compute Q_i; i = 1,2,. . .,m, by the equation (7) as below:

$$Q_i = \upsilon\left(S_i - S^*\right) / \left(S^- - S^*\right) + \left(1-\upsilon\right)\left(R_i - R^*\right) / (R^- - R^*) \quad (7)$$

Where
$S^* = \min_i S_i$, $S^- = \max_i S_i$,
$R^* = \min_i R_i$, $R^- = \max_i R_i$,
ν is introduced as weight of the strategy of ‘‘the majority of criteria” (or ‘‘the maximum group utility”), here suppose that v = 0.5.
Step4: sort the alternatives by the values S, R and Q in decreasing order. Now we have three ranking lists.
Step5: Propose as a compromise solution the alternative A' , which is ranked the best by the measure Q (Minimum) if the following two conditions are satisfied:
C1. Acceptable advantage:

$$Q(A'') - Q(A') \geq 1/(m-1) \quad (8)$$

where A'' is the second alternative in the ranking list by Q; that m is the number of alternatives.

Consider three alternatives with four criteria in table (2):

Table 2 Three alternatives with four criteria

	C_1^-	C_2^+	C_3^+	C_4^+
A_1	5	8	13	4
A_2	4	10	9	2
A_3	8	12	6	3

At first, we scale the data in table (2) and make table (3):

$$n_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}}$$

Table 3 Scaled data in table (2)

	C_1^-	C_2^+	C_3^+	C_4^+
A_1	0.488	0.456	0.769	0.743
A_2	0.390	0.570	0.532	0.371
A_3	0.781	0.684	0.355	0.557

and then we find $f_j^* = \max_j f_{ij}$, $f_j^- = \min_j f_{ij}$ in table (3) and make table (4) :

Table 4 f_j^*, f_j^-

	C_1^-	C_2^+	C_3^+	C_4^+
f_j^*	0.781	0.684	0.769	0.743
f_j^-	0.390	0.456	0.532	0.371

The we use equations (5), (6) and (7)and make table (5) with $w_j = 1$ for $j = 1,2,3,4$ and $\nu = 0.5$:

Table 5 Rank by Q,S and R

Q		S		R	
0	A_1	0.32	A_1	0.299	A_1
0.655	A_3	0.47	A_3	0.305	A_2
0.855	A_2	0.81	A_2	0.336	A_3

C2. Acceptable stability in decision making:
Alternative A′ must also be the best ranked by S or/and R. This compromise solution is stable within a decision making process, which could be ‘‘voting by majority rule” (when ν > 0.5 is needed), or ‘‘by consensus” ν =0.5, or ‘‘with veto”(ν < 0.5). Here, ν is the weight of the decision making strategy ‘‘the majority of criteria” (or ‘‘the maximum group utility”).

If one of the conditions is not satisfied, then a set of compromise solutions is proposed, which consists of:

_ Alternatives A′ and A″ if only condition C2 is not satisfied, or

_ Alternatives A′ , A″ , . . .,$A^{(M)}$ if condition C1 is not satisfied; $A^{(M)}$is determined by the relation $Q(A^{(M)})$ _ Q(A′) < DQ for Maximum M (the positions of these alternatives are ‘‘in closeness”).

The decision maker should accept compromise solution because DM provides a maximum utility of the majority (represented by min S), and a minimum individual regret of the opponent (represented by min R). S and R are integrated into Q for compromise solution, the base for an agreement established by mutual concessions.

4 Problem Definition and the Proposed Method

Hafezolkotob et al. [40] have studied ROBUST DEA with different scenarios by proposing a case study from Electric Energy Distribution Company. These data are used for presenting the method. Firstly, the proposed method in this article which is a combination of data enveloping analysis and VIKOR is presented.

4.1 Method

Suppose that we have n DMUs and t scenarios for the inputs and outputs in which the occurrence probability of each scenario is specified. Consider p_j (j=1,2,…, t) to be the occurrence probability of j^{th} scenario for each DMU, it is clear that $\sum_{j=1}^{t} p_j = 1$.

Using the combination of DEA and VIKOR, now we want to do the ranking of these units with specified probability of different scenarios occurrence. In the table (1), consider A_is as the DMUs and C_js as the n^{th} scenario for each DMU. Then, f_{ij} can be the efficiency score of the i^{th} decision making unit for the j^{th} scenario in the table (1). Therefore, f_{ij} can be easily obtained by using the model (3) for all the DMUs and consequently the table (2) is completed. The decided weighs for the VIKOR method in the table (1) which are w_j are considered as the occurrence probability of each scenario which is p_j. Therefore, the table (6) is compiled as below:

Table 6 Efficiency score of DMUs with n scenarios

	S_1	S_2	$\cdots$	S_n
DMU_1	f_{11}	f_{12}	$\cdots$	f_{1n}
DMU_2	f_{21}	f_{22}	$\cdots$	f_{2n}
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
DMU_m	f_{m1}	f_{m2}	$\cdots$	f_{mn}
	P=[p_1 , p_2, …,p_n]			

In the table (6), S_j (j=1,…,n) is the available scenarios for each decision making unit. Now we can rank the decision making units by using the VIKOR method by the table (6).

We could rank the decision making units using the TOPSIS method. However, considering the article written by Opricovic, Tzeng [41], we decided to do the ranking using the VIKOR method.

4.2 Numerical example

To present the method DEA/VIKOR, a numerical example is given here. Hafezolkotob et al. [40] studied their proposed method named ROBUST DEA by providing a numerical example on electric energy distribution units in Iran with three different scenarios including pessimistic, probable and optimistic each of which has the occurrence probability of 0.25. 0.5 and 0.25 respectively. That problem and its data have been applied in this article. However, the DEA/VIKOR method has been used for ranking them.

There are 38 DMUs in this example in which the inputs are the number of workers, the length of network and the capacity of transformers; and outputs are total selling of power and the number of customers.

We achieved the efficiency scores of the decision making units by applying the model (3) in the table (7) for 38 decision making units which have three scenarios.

Table 7 Efficiency score of DMUs with 3 scenarios

DMUs	S_1	S_2	S_3
DMU1	0.8038	0.7512	0.7137
DMU2	0.3999	0.3738	0.3551
DMU3	0.6989	0.6532	0.6205
DMU4	0.7349	0.6868	0.6205
DMU5	0.861	0.7794	0.7133
DMU6	0.6945	0.6296	0.5773
DMU7	0.5848	0.5385	0.5029
DMU8	0.6123	0.5621	0.523
DMU9	0.6671	0.6131	0.5713
DMU10	0.5403	0.5049	0.4797
DMU11	0.919	0.8589	0.8159
DMU12	0.9813	0.8924	0.8212
DMU13	0.8075	0.7547	0.717
DMU14	0.9724	0.8923	1
DMU15	0.8744	0.8172	0.7763
DMU16	0.7654	0.7153	0.6795
DMU17	0.7573	0.687	0.6305
DMU18	0.8975	0.826	0.7711

DMUs	S_1	S_2	S_3
DMU19	0.826	0.7493	0.6876
DMU20	1	1	0.8198
DMU21	0.5943	0.5373	0.491
DMU22	0.397	0.3601	0.3303
DMU23	0.5498	0.5054	0.4212
DMU24	0.8095	0.7359	0.6769
DMU25	0.6264	0.5678	0.5206
DMU26	0.5917	0.553	0.5253
DMU27	0.7326	0.6846	0.6504
DMU28	0.451	0.4096	0.3764
DMU29	0.5398	0.5045	0.4793
DMU30	0.4889	0.457	0.4341
DMU31	0.5183	0.4706	0.4323
DMU32	0.7507	0.68	0.6228
DMU33	0.7557	0.7063	0.671
DMU34	0.7367	0.6885	0.6541
DMU35	0.7625	0.7085	0.6688
DMU36	0.6421	0.6001	0.5701
DMU37	0.5447	0.4929	0.451
DMU38	0.7259	0.6582	0.6037

We complete the table (7) after achieving the efficiency scores of all the decision making units for different scenarios. Left to right, the first column of the data available is for the efficiency scores of the pessimistic scenario, the second is for probable scenario and the third column is for the optimistic scenario each of which has the occurrence probability of 0.25. 0.5 and 0.25 respectively.

Efficiency graphs can be drawn as below for DMUs based on their achieved efficiency score.

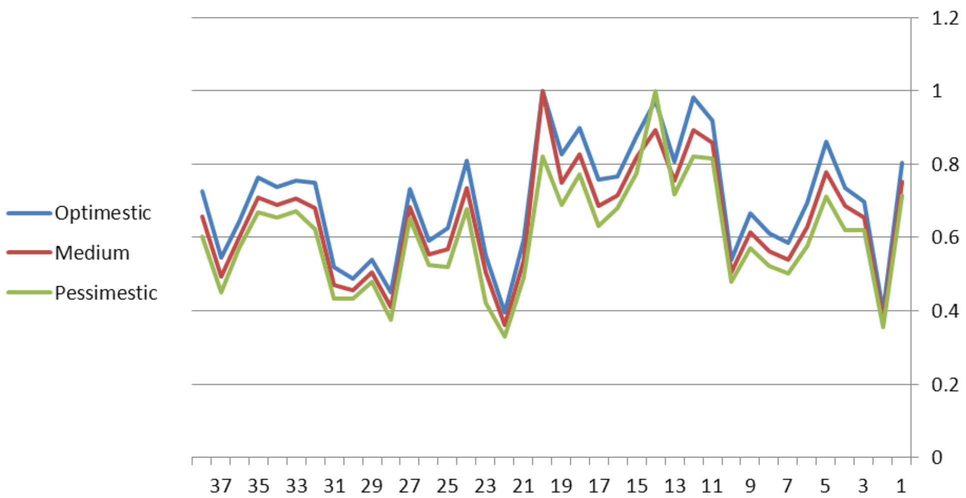

Fig. 1 Efficiency graphs for DMUs with 3 scenarios

Regarding the figure (1) and the efficiency function of each scenario, we cannot achieve an appropriate ranking based on their efficiency score. As seen in the efficiency graph in the figure (1), the optimistic graph is not always the best and the pessimistic mode is not always the worst. Similarly, the medium mode is not always placed between their efficiency graphs.

If we calculate the weighted average of efficiency graphs and consider the weights to be the occurrence probability of each scenario, we have the figure (2).

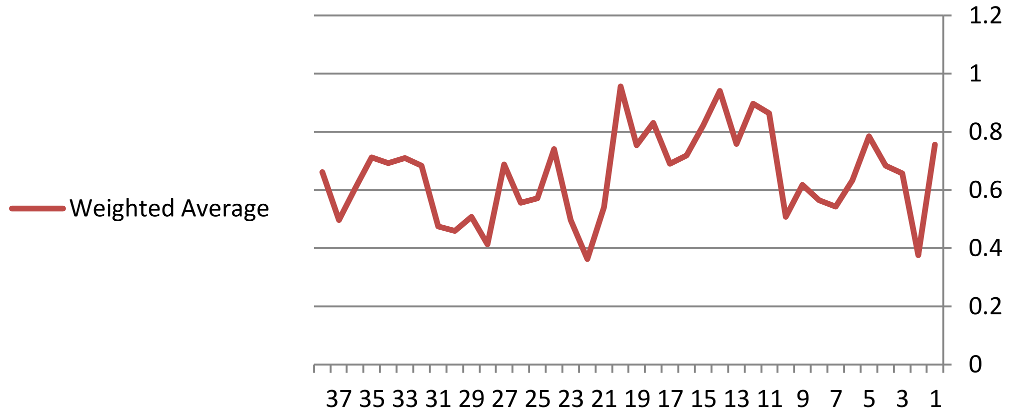

Fig.2 weighted average graph

Regarding the weight of the middle scenario, after calculating the weighted average, efficiency graph is much more similar to medium mode. In other words, optimistic and pessimistic modes are not used at all in calculating the efficiency score. Therefore, the weighted average method is not an appropriate method for calculating the efficiency score of decision making units with several scenarios.

Now we want to achieve the ranking of the decision making units with different scenarios and specified occurrence probability by using the proposed method in 4-1 and a combination of the DEA and VIKOR methods. As mentioned before, occurrence probability of the scenarios can be considered as weights in VIKOR method. Applying the VIKOR method for the data in table (7) and make the table (8):

Table 8 Q, S, R by VIKOR

DMU	Q_i	S_i	R_i
DMU1	0.31595	0.382624873	0.194405376
DMU2	0.976285	0.978835004	0.489295202
DMU3	0.487441	0.537481914	0.270979841
DMU4	0.435031	0.496302439	0.244725738
DMU5	0.266045	0.337024741	0.172370683
DMU6	0.528256	0.573873132	0.289420222
DMU7	0.687931	0.718310688	0.360603219
DMU8	0.646605	0.68096562	0.342162838
DMU9	0.557354	0.600365447	0.302312861
DMU10	0.746873	0.771674804	0.386857321
DMU11	0.127548	0.212558494	0.110251602
DMU12	0.068365	0.158574843	0.084075637
DMU13	0.309841	0.377124179	0.191670574
DMU14	0.034695	0.09559656	0.084153774
DMU15	0.2005	0.278415327	0.142834818
DMU16	0.378777	0.439363439	0.222456634

DMU	Q_i	S_i	R_i
DMU17	0.427787	0.483126251	0.244569464
DMU18	0.184776	0.263903306	0.135958744
DMU19	0.31875	0.384648668	0.195889983
DMU20	0	0.067268926	0.067268926
DMU21	0.689787	0.719751982	0.361540866
DMU22	1	1	0.5
DMU23	0.755807	0.789183614	0.386466635
DMU24	0.342269	0.405954176	0.206360369
DMU25	0.636418	0.671561951	0.337709017
DMU26	0.662751	0.695758143	0.349273324
DMU27	0.432466	0.487813309	0.246444757
DMU28	0.913349	0.921724824	0.461322082
DMU29	0.747593	0.77234397	0.38716987
DMU30	0.830731	0.84743519	0.424285045
DMU31	0.806582	0.825291418	0.413658384
DMU32	0.440046	0.494206595	0.250039069
DMU33	0.394529	0.453590409	0.229488983
DMU34	0.42566	0.481684908	0.243397406
DMU35	0.39055	0.449873417	0.227769964
DMU36	0.580332	0.621336089	0.312470699
DMU37	0.767499	0.789940809	0.396233787
DMU38	0.478193	0.528652489	0.26707298

Table 9 Rank by Q, S, R

Rank	Q	S	R
1	DMU 20	DMU 20	DMU 20
2	DMU 14	DMU 14	DMU 12
3	DMU 12	DMU 12	DMU 14
4	DMU 11	DMU 11	DMU 11
5	DMU 18	DMU 18	DMU 18
6	DMU 15	DMU 15	DMU 15
7	DMU 5	DMU 5	DMU 5
8	DMU 13	DMU 13	DMU 13
9	DMU 1	DMU 1	DMU 1
10	DMU 19	DMU 19	DMU 19
11	DMU 24	DMU 24	DMU 24
12	DMU 16	DMU 16	DMU 16
13	DMU 35	DMU 35	DMU 35
14	DMU 33	DMU 33	DMU 33
15	DMU 34	DMU 34	DMU 34
16	DMU 17	DMU 17	DMU 17
17	DMU 27	DMU 27	DMU 4
18	DMU 4	DMU 32	DMU 27
19	DMU 32	DMU 4	DMU 32
20	DMU 3	DMU 3	DMU 38
21	DMU 38	DMU 38	DMU 3

Rank	Q	S	R
22	DMU 6	DMU 6	DMU 6
23	DMU 9	DMU 9	DMU 9
24	DMU 36	DMU 36	DMU 36
25	DMU 25	DMU 25	DMU 25
26	DMU 8	DMU 8	DMU 8
27	DMU 26	DMU 26	DMU 26
28	DMU 7	DMU 7	DMU 7
29	DMU 21	DMU 21	DMU 21
30	DMU 10	DMU 10	DMU 23
31	DMU 29	DMU 29	DMU 10
32	DMU 23	DMU 23	DMU 29
33	DMU 37	DMU 37	DMU 37
34	DMU 31	DMU 31	DMU 31
35	DMU 30	DMU 30	DMU 30
36	DMU 28	DMU 28	DMU 28
37	DMU 2	DMU 2	DMU 2
38	DMU 22	DMU 22	DMU 22

Table 10 Ranked by VIKOR method

Rank	DMU
1	DMU20
2	DMU14 - DMU12
3	DMU11
4	DMU18 - DMU15
5	DMU5
6	DMU13 - DMU1 - DMU24 - DMU19
7	DMU16 - DMU35 - DMU33
8	DMU34 - DMU17 - DMU 27 - DMU 4 - DMU32
9	DMU3 - DMU38 - DMU6
10	DMU9 - DMU36
11	DMU25 - DMU8 - DMU26 - DMU7 - DMU21
12	DMU10 - DMU29 - DMU23 - DMU37 - DMU31 - DMU30
13	DMU28 - DMU2 - DMU22

According to the article written by Opricovic, Tzeng [41], it is decided to do the ranking by using the VIKOR method rather than TOPSIS method after achieving the efficiency score of each decision making unit with different scenarios.

5 Conclusions

Not so much research has been conducted on data with different scenarios which have specified occurrence probability and the proposed methods have difficult models for solving the problem and ranking the decision making units. Furthermore, infeasibility is probable in the proposed models. These models ignore the infeasibility supposition in order to rank which is not a true criterion for ranking the decision making units. In this research, we believe that using the data enveloping analysis and Vikor method, we have proposed a method for ranking the units which can rank the decision making units with data which have different scenarios without suffering from the difficulties of the previously proposed models

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