International Journal of Applied Operational Research
Vol. 7, No. 4, pp. 21-44, Autumn 2017

Journal homepage: ijorlu.liau.ac.ir

Using metaheuristic algorithm to solve a multi objective portfolio selection problem: application in renewable energy investment policy

N. Manavizadeh*, M.A. Yavari, H. Farrokhi-Asl

Received: 9 May 2017 **;** **Accepted:** 20 September 2017

Abstract Energy is one of the key factors in economic and satisfaction of energy demand is an indicator to show economic growth and community development. Renewable energy sources are desirable alternatives for conventional energies due to their advantages such as less pollutant and job generation growth. Hence, governments try to stimulate investors and non-government organizations to invest in renewable energy projects. In this study, a multi-objective mathematical model is proposed to determine the optimal portfolio for financing projects of renewable energies. The model aims to minimize the weighted cost of capital of the investors and to minimize greenhouse gas emissions. On the other hand, the model maximizes net present value and job generation for urban, rural, and remote areas. Bonds, common stocks, and bank loans are three possible ways to cover the required budget. The small size of the problem is solved exactly using GAMS 22.9 software. Since the non-deterministic polynomial-time hard nature of the problem, fast non-dominated sorting genetic algorithm is applied as a meta-heuristic solution approach to solve the large sized problems. The obtained results show the superiority of bonds among other capital sources. Moreover, we conclude that photovoltaic is the most attractive renewable source for electricity generation.

Keyword: Portfolio Selection Problem; Engineering Economic; Renewable Energy Sources; Greenhouse Gases; Meta-Heuristic Algorithms.

1 Introduction

Energy is one of the key economic elements of a nation; hence, satisfaction of energy demand is an essential issue that should be considered to provide economic growth and consequently community development. According to the available statistics, the trend of energy demand shows a willingness to continue increasing in the future [1]. In such a situation, conventional energy consumption (i.e. fossil fuels) is growing rapidly. Although conventional energy

* **Corresponding Author. (✉)**
E-mail: n,manavi@khatam.ac.ir (N.Manavizadeh)

N. Manavizadeh

Department of Industrial Engineering, KHATAM University, Tehran, Iran

M.A. Yavari

Department of Industrial Engineering, KHATAM University, Tehran, Iran

H. Farrokhi-Asl

School of Industrial Engineering, Iran University of Science & Technology, Tehran, Iran

[Downloaded from ijaor.ir on 2026-11-28]

resources are interesting due to their low capital requirements, they are perishable and emit a large amount of greenhouse gas (GHG). It is the main reason for global warming and climate change that concern environmentalists [2]. In the United Kingdom, the Department of Environment, Food and Rural Affairs (DEFRA) uses a different measure called the shadow price of carbon and suggests to set it at £27/t of CO2 emitted in the year 2010, and to increase it by 2% for each subsequent year [3]. Increasing usage of these sources of energy causes rapid depletion of these resources. Hence, governments try to stimulate investors and non-government organizations to invest in renewable energy projects. According to this fact, we can save some expenditure and achieve more profit and healthy environment by means of good management of energy systems. In this paper, we suggest a multi-objective mathematical model in order to determine the optimal portfolio for financing projects of renewable energies, which are more compatible with the environment. The model also addresses the sustainability concept by considering of economic, social, environmental, and technical objectives.

Renewable energy sources (RES) are suitable substitutes for the conventional kinds of energy resources since they are known to produce much less amounts of greenhouse gas. Furthermore, renewable energy development would help to supply the energy demand in rural and remote areas sustainably. In addition, it creates benefits e.g. employment generation that leads to reduction of migration towards urban areas [4]. There are several different types of these energy sources (e.g. wind power, hydropower, geothermal, photovoltaic, biomass, etc.). Most of these renewable ones are available in different areas around the world. The use of renewable energy sources for electricity generation is rapidly growing around the world. Nowadays, these sources contribute a significant amount of the energy portfolio in developed countries. Hence, in the energy planning field, it seems vital to analyze investment on RES, considering economic, environmental, and social aspects simultaneously to establish a sustainable energy system.

In this study, four sustainable indicators are proposed in the renewable energy investment to determine an optimal combination of renewable energy technologies in urban, rural, and remote areas. These indicators consist of net present value (NPV), greenhouse gases emission, employment generation, and cost of capital (CC).

First, in every investment project, investors are willing to determine the difference between the present values of cash inflows and outflows that are called net present value. From an economic viewpoint, a qualified energy portfolio is the one maximizing the NPV of the cash flows along the time. It is essential to invest in renewable energy projects in which the NPV would be maximized. Secondly, renewable energy sources do not have greenhouse gas emissions during their operation; on the other hand, they may emit large amounts of GHG during their whole life cycle. Thus, from an environmental viewpoint, it is necessary to invest in renewable energy sources considering mitigation of GHG emissions. Moreover, as an aforementioned, investment in renewable energy projects in different areas would generate job opportunities; hence, maximization of employment generation should be considered while renewables investment planning. It is an important economic and social factor among nations. Finally, a required budget for investment in renewable energy projects can be provided through different financial sources such as selling bonds, selling common stocks and borrowing from banks. The CC shows the expectations of external investors that must be satisfied via obtained revenues from the projects. Thus, it is essential to determine the best combination of funding for each project in order to find the optimal portfolio minimizing a weighted cost of capital.

[Downloaded from ijaor.ir on 2026-11-28]

In this research, a multi-objective mathematical formulation is developed to gather the four objectives. A major complexity is that the objectives described above are conflicting. For example, the less greenhouse gas emitted in a renewable technology, the less NPV may result. Therefore, the weighted sum method (WSM) is applied to deal with the problem of existing inconsistency. This solution approach integrates objective functions considering a proper weight for each of them that shows the preferences of decision makers and provides Pareto optimal solutions. When the size of the problem increases, it is not practical to use exact solution methods, because a computational time for solving the model increases, exponentially. Consequently, for medium and large sizes, fast non-dominated sorting genetic algorithm (NSGA-II) is applied as a meta-heuristic solution approach, which prepares desirable near optimal solutions in a much less time.

The remainder of the paper is organized as follows: in Section 3, the relevant previous works are reviewed. In Section 4, the problem is defined in details and the mathematical formulation of the proposed model is presented. Section 5 describes the proposed solution algorithms, then in Section 6 numerical experiments are conducted for small, medium, and large sizes problems to show the efficiency of the proposed model. Finally, conclusions, remarks and future research directions are provided in Section 7.

2 Review of literature

Renewable energy sources have been widely proposed in the literature due to their important economic, environmental, and social impacts and their rapid growth in energy systems. Pantaleo et al. [5] discussed the technical and economic feasibility of offshoring wind farms for four different locations in the Puglia region. For discussing the economic feasibility, the cost of energy, calculated by the leveled cost of energy (LPC) and the profitability was evaluated by the NPV and the internal rate of return (IRR). Ozerdem et al. [6] discussed the technical and economic feasibility of wind power in Izmir, Turkey. In technical assessment, speed of wind, prevailing wind direction and temperature measurements are considered. In economical appraisal three scenarios, including auto-producer, auto-producer group and independent power producer (IPP) are evaluated and NPV, IRR, and payback period (PBP) are applied to compare the scenarios. Kahraman et al. [7] used fuzzy multi criteria decision-making approach to the problem of selecting the most appropriate renewable energy source in Turkey. They applied fuzzy axiomatic design (AD) and fuzzy analytic hierarchy process (AHP) to evaluate five different types of renewable ones under four main criteria. Akdag and Guler [8] perused the situation of wind energy as a renewable energy source and its development around the world and then in Turkey. They analyzed the cost of electricity generation via wind power in distinct locations of Turkey. The obtained results delineated that it is feasible and cost effective to generate wind electricity in the supposed locations.

Yang et al. [9] executed an economic analysis of a wind firm by applying three alternatives (cost benefit analysis of current situations, government wind power subsidy on the wind power price and clean development mechanism (CDM) of wind farms) which were appraised with respect to three economic metrics (i.e. NPV, IRR, and payback period). Trapani et al. [10] have comparatively evaluated the utilization of offshore photovoltaic (PV) systems. This paper focused on crystalline PV panels because they are used in pontoon models. The authors proposed a flexible thin film design for offshore PVs that hovers on water and discussed its strengths and weaknesses. Liu et al. [11] identified the gray sustainability indicator to measure the sustainability of a renewable energy system

[Downloaded from ijaor.ir on 2026-11-28]

considering eleven economic, environmental, and social assessment criteria. As a real case study, they applied this indicator for four renewable energy systems with different combinations of grid, solar PV and wind energy in Australia.

From an environmental perspective, pollutant emission is an inevitable concern; hence, the problem of mitigating pollutant emissions (especially CO2) has been discussed widely in the literature of energy. Hashim et al. [11] used a mixed integer linear programming (MILP) model for the problem of energy planning optimization with regard to CO2 emissions mitigation. They analyzed an Ontario Power Generation (OPG) fleet from the perspective of three modes: (1) economic mode, (2) environmental mode, and (3) integrated mode that considers two prior modes. The authors found that fuel balancing and fuel-switching options are effective ways for CO2 emissions reduction. Varun et al. [1] evaluated four different energy sources including wind, solar PV, solar thermal, and small hydro on the basis of sustainability indicators They considered the costs of producing electricity by using renewable energy systems, the energy pay-back time (EPBT), as well as greenhouse gas emission. Li et al. [12] presented an integrated fuzzy-stochastic optimization model (IFOM) for planning the regional energy system in relation to GHG mitigation. The paper applied uncertainties such as probability distributions, fuzzy-intervals and their mixtures. The model was used to solve a durable planning of a regional energy system with two objectives of sustainability and safety of supply considering six technologies (i.e. coal, natural gas, hydro, wind, solar and nuclear). Some authors proposed CO2 emissions, employment generation, and investment in renewable energy sources. Frondel et al. [13] investigated the impacts of the government policies on the situation of the renewable energy sources act of Germany from the perspective of its total costs, employment generation, and climate change.

Kazemi and Rabbani [14] proposed a multi-objective linear programming for decentralized energy planning with considering demand-side management and environmental measures. In this paper, five strategies (DSM, PV, wind, hydro, and geothermal) were evaluated against various sustainability indicators (SIs) such as electricity generation cost, job creation, water consumption, GHG emissions, and land use requirements. Solving the model showed that in the optimum manner hydro placed in the first place. Masini and menichetti [15] discussed about key factors that play an effective role in the context of the renewable energy investment. They developed a two-stage conceptual framework model considering a wide range of investors (not only venture capitalists). In the first stage, they examined if human behavioral factors have a measurable effect on the renewable energy investment projects. In the second stage, they evaluated the impact of the renewable energy share in the portfolio and the investors' attitude towards technological risks on the performance of investment. Herran and Nakata [16] developed a linear programming (LP) mathematical model in order to optimally design a decentralized energy system for electricity generation using local biomass resources in rural areas. The performance of the designed systems was evaluated from three aspects: 1) total net cost, 2) local net income, 3) CO2 emissions. The obtained results represented that in the case of generating electricity using local biomass, the cost of electricity generation decreases and local net income increases.

[Downloaded from ijaor.ir on 2026-11-28]

Table 1 An overview of the previous studies

Author(s)		[5]	[6]	[7]	[9]	[17]	[11]	[18]	[19]	[20]	[21]	[22]	[16]	[14]	[13]	Current study
Economic indicators	NPV	*	*		*											*
	IRR	*	*		*											
	PBP		*		*											
	CC															*
Environmental indicators	Pollutant emissions			*	*	*	*	*	*	*	*	*	*	*	*	*
	Water consumption													*		
	Land use													*		
Social indicators	Job creation					*				*		*		*	*	*
	Social acceptability			*						*		*				
Technical indicators	reliability					*										
	Availability	*				*								*		*
Model	Single objective				*											
	MADM			*												
	MODM			*				*	*							*
Solution method	Exact					*	*					*	*	*		*
	Meta-heuristic										*					*

As shown in Table 1, a few papers in the literature have taken a combination of economic, social, environmental, and technical objectives into consideration. Although financing investment projects by means of proper and rational financial sources is a great economic concern, determining the best portfolio for funding investment in renewable energy projects have not been proposed in former studies. To the best of our knowledge, it is the first time that determining the optimal portfolio for investment projects is investigated along with other sustainable indicators in the field of renewable energy technologies.

3 Problem definition

In this section, we formulate the multi-period problem of investment in power plant foundation projects for electricity generation in different areas using renewable energy sources. Like other investment problems, it is necessary to determine whether a foundation project is economically feasible or not. Here, the NPV method is applied for this purpose to select projects that maximize the net present value of the cash flows for investors. Besides, two sustainability indicators are considered as two distinct objective functions: minimization of greenhouse gas emissions due to renewable energy sources and maximization of employment generation due to power plant foundation projects.

In order to cover the required budget for each period, we utilize three ways: (1) selling bonds, (2) selling common stocks, (3) bank loans. For each of these ways, the CC which

[Downloaded from ijaor.ir on 2026-11-28]

shows the expectations of the people who purchase bonds and stocks, and banks, should be determined. The objective is to minimize the weighted cost of capital to find the optimal portfolio for the investment in every period. We should decide on an amount of the generated energy by means of each type power plant in areas for available periods. Each established power plant requires investment for construction. We also should determine portion of each source of capital in constructing power plant. Some parameters such as cost of capital, operational cost for each energy source, the installation cost of power plant and so forth effect on our decisions. The following mathematical formulation gives insight about the relationship between these decisions and parameters.

3.1 Assumptions

All assumptions considered in the mathematical model are as follows:

- Renewable energy projects are implemented to generate electricity in rural, urban, and remote areas.
- Electricity demand for each area in every period is known and deterministic.
- Four different renewable energy sources are proposed: wind power, hydropower, geothermal and photovoltaic.
- The availability of each renewable source in the areas is deterministic and finite.
- Greenhouse gas emissions and employment generation of each renewable energy source are known and deterministic.
- The required budget for the power plants' foundation is procured through bonds, common stocks, and bank loans.
- The cost of capital for common stocks is calculated by Gordon-Shapiro growth model (i.e. $CC = \frac{D_e}{P_e} + \frac{E_e - D_e}{BV_e}$).
- A specific percentage of the total benefit in every period related to the sold common stocks is paid to the propertied persons and the rest is invested in the projects with a specific rate for the next periods.
- The extra budget remained in every period is invested with a specific rate for the next period.
- The annual payments on a bank loan are considered equal.

3.2 Mathematical formulation

Indices:

i set of renewable energy resources
j set of areas
t set of periods
m set of sources of capital

Parameters:

P_{ijt} The installation cost of power plant of *i*th energy source in *j*th area in period *t*;

M_{ijt} Maintenance cost of power plant of *i*th energy source in *j*th area in period *t*;

[Downloaded from ijaor.ir on 2026-11-28]

O_{ijt} Operational cost of power plant of *i*th energy source in *j*th area in period *t*;

Cap_{ijt} Electricity generation capacity of power plant of *i*th energy source in *j*th area in period t;

D_j Energy demand of *j*th area;

A_i Availability of the *i*th energy source;

η_{ij} Conversion efficiency for the *i*th energy source for *j*th area;

C_{ijt} Electricity generation cost for per unit of energy generated by power plant of *i*th energy source in *j*th area in period *t*;

I_j Revenue obtained from selling per unit of generated energy in *j*th area;

$Ipay_t$ Interest payments for the loan in period *t*;

Pay_t Payments of the loan in period *t*;

T_e Effective tax rate;

GHG_i Greenhouse gas emissions due to per unit of generated energy by power plant of *i*th energy source;

EG_i Number of generated jobs due to per unit of generated energy by power plant of *i*th energy source;

B_{jt} Required budget for investment in *j*th area in period *t*;

CC_{1ijt} Cost of capital of bonds for power plant of *i*th energy source in *j*th area in period *t*;

CP_b Number of interest payments of bonds per year;

i_{lb} The nominal annual interest rate for bonds;

$P_{e_{ijt}}$ Market value of per share of common stocks for power plant of *i*th energy source in *j*th area in period *t*;

$E_{e_{ijt}}$ Earnings of per share of common stocks for power plant of *i*th energy source in *j*th area in period *t*;

$D_{e_{ijt}}$ Dividends paid per share of common stocks for power plant of *i*th energy source in *j*th area in period *t*;

$BV_{e_{ijt}}$ Book value of each share of common stocks for power plant of *i*th energy source in *j*th area in period *t*;

l The rate of book value that determines the earnings of per share of common stocks;

CC_{2ijt} Cost of capital of common stocks for power plant of *i*th energy source in *j*th area in period *t*;

b Fraction of earnings that is retained by the firm;

$n_{e_{ijt}}$ Number of common stocks sold for power plant of *i*th energy source in *j*th area in period *t*;

[Downloaded from ijaor.ir on 2026-11-28]

i_{lo} The nominal annual interest rate for loans;

CC_3 Cost of capital of loans;

CP_{lo} Number of interest payments of loans per year;

N_{lo} Duration of loan payments;

ii The annually effective interest rate;

k The annually effective interest rate for investment of extra money;

Continuous decision variables

x_{ijt} Amount of generated energy by means of power plant of *i*th energy source in *j*th area in period *t*;

lot_{mijt} Portion of *m*th source of capital in constructing power plant of *i*th energy source in *j*th area in period *t*;

Integer decision variables

$z_{ijt} = \begin{cases} 0 \\ 1 \end{cases}$ If power plant of *i*th energy source is installed in *j*th area in period *t*, it is 1;

$y_{mijt} = \begin{cases} 0 \\ 1 \end{cases}$ If *m*th capital source is used for power plant of *i*th energy source in *j*th area in period *t*, it is 1;

Objective functions:

f_1: Maximize

$$\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{t=1}^{T}[z_{ijt}(-P_{ijt}+D_{jt}\times I_j-(M_{ijt}+O_{ijt}+C_{ijt}\times x_{ijt}+Ipay_t))(1-T_e)-Pay_t+S_{j(t-1)}(1+k))(\frac{P}{F},ii\%,t)$$

$$+(y_{2ijt}\times n_{e_{ijt}}\times(E_{e_{ijt}}-D_{e_{ijt}})(\frac{F}{P},k\%,(T-t))(\frac{P}{F},ii\%,T))] \quad (1)$$

f_2: Minimize $\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{t=1}^{T} z_{ijt}\times GHG_i\times x_{ijt}$ (2)

f_3: Maximize $\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{t=1}^{T} z_{ijt}\times EG_i\times x_{ijt}$ (3)

f_4: Minimize $\frac{1}{\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{m=1}^{M}\sum_{t=1}^{T} lot_{mijt}\times y_{mijt}}\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{m=1}^{M}\sum_{t=1}^{T} lot_{mijt}\times CC_{mijt}\times y_{mijt}$ (4)

s.t.

$\sum_{i=1}^{I}\sum_{t=1}^{T} z_{ijt}\times x_{ijt}\geq D_j \quad \forall j$ (1)

$\sum_{j=1}^{J}\sum_{t=1}^{T}\frac{z_{ijt}\times x_{ijt}}{\eta_{ij}}\leq A_i \quad \forall i$ (2)

$B_{jt}=\sum_{i=1}^{I}\frac{z_{ijt}\times P_{ijt}}{Cap_{ijt}} \quad \forall j,t$ (3)

[Downloaded from ijaor.ir on 2026-11-28]

$$\sum_{i=1}^{I}\sum_{m=1}^{M} lot_{mijt} \times y_{mijt} + S_{j(t-1)} \geq B_{jt} \qquad \forall j,t \qquad (4)$$

$$S_{jt} = \sum_{i=1}^{I}\sum_{m=1}^{M} lot_{mijt} \times y_{mijt} - B_{jt} \qquad \forall j,t \qquad (5)$$

$$CC_{1ijt} = [(1+\frac{i_{lb}}{CP_b})^{CP_b} - 1] \times (1-T_e) \qquad \forall i,j,t \qquad (6)$$

$$E_{e_{ijt}} = l \times BV_{e_{ijt}} \qquad \forall i,j,t \qquad (7)$$

$$D_{e_{ijt}} = b \times E_{e_{ijt}} \qquad \forall i,j,t \qquad (8)$$

$$n_{e_{ijt}} = \frac{lot_{2ijt} \times y_{2ijt}}{BV_{e_{ijt}}} \qquad \forall i,j,t \qquad (9)$$

$$CC_{2ijt} = \frac{D_{e_{ijt}}}{p_{e_{ijt}}} + \frac{E_{e_{ijt}} - D_{e_{ijt}}}{Bv_{e_{ijt}}} \qquad \forall i,j,t \qquad (10)$$

$$CC_{3ijt} = [(1+\frac{i_{lo}}{CP_{lo}})^{CP_{lo}} - 1](1-T_e) \qquad (11)$$

$$Pay_t = \frac{\sum_{i=1}^{I}\sum_{j=1}^{J} lot_{3ijt} \times y_{3ijt}}{N_{lo}} \qquad \forall t \qquad (12)$$

$$Ipay_t = (\sum_{i=1}^{I}\sum_{j=1}^{J} lot_{3ijt} \times y_{3ijt} - Pay_t) \times i \qquad \forall t \qquad (13)$$

$$n_{ijt} \in N \cup \{0\}$$

$$z_{ijt}, y_{ijt} \in \{0,1\}$$

$$x_{ijt}, lot_{mijt} \geq 0 \qquad \forall i,j,t \qquad (14)$$

The first objective function maximizes the net present value of the cash flow for investment in renewable energy power plant foundation projects. The first part considers all incomes and outcomes of the selected projects and converts the net after tax cash flow to the present value. The amount of money, which is more than the required budget, remains at the end of each period ($S_{j(t-1)}$). This extra money is invested with the specific rate in the next period and will be used in the portfolio of later periods. The second part shows the amount of net profit, which is retained and not paid to the propertied people. The money preserved at the end of each period is invested with the specific rate for the next periods. The objective function 2 minimizes the total amount of greenhouse gas, which is emitted due to renewable energy investment projects. The objective function 3 considers the number of jobs created due to foundation projects and maximizes employment generation. The fourth objective function minimizes the weighted cost of capital that consists of the cost of capital for the bonds, common stocks, and bank loans that are the sources for procurement of the required capital for electrifying considered areas.

[Downloaded from ijaor.ir on 2026-11-28]

Equation (1) indicates that the demand for electricity in each area should be supplied through energy generation by renewable energy power plants. Equation (2) shows the limitation of renewable resources availability. Equation (3) determines the required budget in every period. Equation (4) indicates that the amount of money obtained through capital sources in every period, plus the extra budget remained from the ex-period, should be at least equal to the required budget of that period. Equation (5) calculates the extra money remained at the end of each period in each area, which is the abstraction of the total money, obtained through capital sources in each period and required a budget. Equation (6) specifies the cost of capital of bonds. Equations (7) and (8) calculate the amount of earnings for each share and the portion of earnings that is paid to one share of the common stocks at period t, respectively. Equation (9) determines the book value for per share of common stocks. In equations (10) and (11), the cost of capital for common stocks and bank loans are calculated respectively. Equations (12) and (13) specify the annual payments and annually interest payments that should be paid for bank loans. The type of all decision variables is determined in Equation (14).

In this paper, we consider a Capacitated Inventory Routing Problem (CIRP) for perishable products by considering environmental aspects where a set of heterogeneous vehicles with different levels of technologies is used. As more a vehicle's level of technology, it has more transportation costs, which could be related to the kind of maintenance, should be done for it and less environmental costs which makes the usage of them reasonable. The network consists of a single depot and a set of different retailers. Each retailer has demands for some different products, which are predictable through historical data. As such, we assume that the demands are deterministic which reasonably does not effect on generality and realism of the problem. As an aforementioned, products that are planned to be transferred are perishable. It means that as product's age increases, its usefulness decreases. Therefore, by passing time, demand of a product reduces from nominal demand. It shows dissatisfaction of customers. We propose a model to schedule an optimal timetable for a multi-period time horizon. All nominal demands should be met. Nevertheless, by passing time, demands may be considered as a lost sale that has a cost for unsold and useless products. Retailers can hold inventory for some days up to their volume capacity of inventory, which makes products aged and reduce their demands. Demands of products may decrease linearly based on [13]. In our proposed model, constraints like sub-tour elimination, i.e., each vehicle should start at a depot and finished its route in the same depot is representative for a closed loop supply chain. Each retailer should be serviced only by one vehicle and one time at each period. Our aim is preparing a timetable for all vehicles to satisfy all retailers' needs and in addition to transportation and inventory costs trying to reduce harmful environmental aspects of transportation and try to increase customer satisfaction by providing appropriate products that should utilities fresh.

4 Methodology

The proposed model is a multi-objective type with conflicting objective functions. For a single objective model, the optimal solution exists in which the objective function has its best value, while generally in multi-objective programming, no one optimal solution can be found to simultaneously optimize all the conflicting objectives. Hence, we should search the feasible decision space to look for solutions that satisfy objective functions altogether. These most

[Downloaded from ijaor.ir on 2026-11-28]

preferred solutions are called Pareto optimal, which are solutions that cannot improve in one objective function without detracting their performance in at least one of the rest [23].
To solve the problem with GAMS 22.9 software, the weighted sum method (WSM) is used to obviate the problem of existing more than one objective function and make it possible to solve the model. In this method, individual objective functions are incorporated to form a single objective in which every function has a proper positive weight that shows the decision maker's preferences. It should be noted that before summing the weighted objective functions, they must be transformed to a comparable unit through normalization. For this purpose, each function is subtracted from its optimal value and then divided by the same. Using the WSM approach, the proposed model in the prior section is reformulated as follows:
Objective function:
Minimize

$$f = w_1\left(\frac{f_1^+ - f_1}{f_1^+}\right) + w_2\left(\frac{f_2 - f_2^+}{f_2^+}\right) + w_3\left(\frac{f_3^+ - f_3}{f_3^+}\right) + w_4\left(\frac{f_4 - f_4^+}{f_4^+}\right) \quad (15)$$

s.t.
Equations (1-14)

$$\sum_{n=1}^{4} w_n = 1 \quad \& \quad w_n \geq 0 \quad \forall n \quad (16)$$

Where w_n and f_n^+ are the weight and the optimal value of *n*th objective function, respectively. Equation (15) indicates that summation of the weights should be equal to one and each weight must be positive.
When the size of the problem increases, exact solution methods are not able to find Pareto optimal set. In this case, meta-heuristic algorithms such as genetic algorithms (GA) are suitable solution methods which are applied widely for multi-objective optimization problems [25]. According to the [26], using a metaheuristics is justified in the following conditions:

- An easy problem with large sized instances
- An easy problem with limited available time
- An NP-hard class of problems
- Optimization with time consuming objective function and constraints for example non-linear ones

Regarding aforementioned conditions and the nature of the proposed model (non-linear and NP-hardness) we decide to apply metaheuristic approach namely NSGA-II.

GA has been the most popular meta-heuristic approaches exploited for multi-objective design and optimization problems, since most multi-objective GA do not require the user to prioritize, scale, or weigh objectives [26]. John Holland [27] first introduced this algorithm. GA is an intelligent probabilistic search algorithm that works by preserving and adapting the characteristics of a set of trial solutions over a number of solutions. In GA terminology, each individual solution is represented by a string, which is referred to as a chromosome and includes a set of discrete units called genes. Genetic algorithm starts with an initial randomly generated population that will be improved by passing through the next iterations using GA's operators (i.e. crossover and mutation). In a crossover, two chromosomes called parents are selected usually with a preference towards their fitness function values. Then the selected parents are mixed to make new chromosomes called offspring. It is expected that offspring inherits good genes of the parents, which make the parents fitter, hence, it is expected that good chromosomes appear more frequently in the population by iteratively using the

[Downloaded from ijaor.ir on 2026-11-28]

crossover, which leads to an overall good solution. In mutation, a number of genes (determined by a mutation rate) in a parent are randomly selected to be changed and make new chromosomes. Mutation brings diversity in the search which helps escape from local optimum solution.

A significant number of developed GA are proposed in order to solve multi-objective problems. In this study, fast non-dominated sorting genetic algorithm (NSGA-II) which was introduced by Deb et al. [28] is deployed to solve the medium and big sizes of the model. NSGA-II is the modified version of NSGA, which eliminates some shortcomings of the former approach. It reduces the computational complexity of non-dominated sorting and enhances elitism (selection of best solutions) which helps preventing the loss of good solutions once they are discovered and accelerates performance of the genetic algorithm remarkably [28]. Pseudo code of NSGA-II is shown in Figure 1.

```
Initialize population
   Generate random population- size npop
   Evaluate objective values
   Generate child population
      Binary tournament selection
      Recombination and mutation
For i=1 to npop
   With parent and child population
       Assign rank based on Pareto dominance
       Loop by adding solutions to next generation starting from the first rank
until M individuals found
       Determine crowding distance between points of each front
   Select points on the lower front and are outside a crowding distance
   Create next generation
       Binary tournament selection
       Recombination and mutation
   Increment generation index
End loop
```

Fig. 1 Pseudo code of NSGA-II

4.1 Proposed NSGA-II

Since the beginning of the nineteenth century, interest concerning multi objective problems (MOPs) area with Pareto approaches has always grown. Population based metaheuristics like NSGA-II seem especially suitable to solve MOPs, since they deal simultaneously with a set of solutions that allow to find several members of the Pareto optimal set in a single run of the algorithm. Moreover, Pareto population based metaheuristics are less sensitive to the shape of the Pareto front [27]. Because of the NP-hard nature of the problem [15], GAMS software is unable to tackle the large size problem. For this reason, we apply a well-known metaheuristic algorithm, namely NSGA-II, to solve the problem. One of the most popular evolutionary multi-objective algorithms is the NSGA-II algorithm. NSGA-II shows good performance in this field and this fact motivates us to use this algorithm.

[Downloaded from ijaor.ir on 2026-11-28]

The main structure of the proposed fast non-dominated sorting genetic algorithm is illustrated in Figure 2. The steps of the algorithm are as follows [29]:

Step 1. A randomly initialized parent population P_0 is generated (*nPop*=100). Each parent is a chromosome that is composed of some genes. In the proposed problem, three dimensions (3D) and four dimensions (4D) chromosomes are defined as parent solutions. In 3D chromosome, when a cell contains a value more than zero at a specific period, it shows the amount of renewable energy source i that is assigned to be invested in area j i.e. x_{ijt}. In 4D chromosome, when a cell contains a value more than zero at a specific period, it shows the amount of capital source m that is assigned to area j for investment in renewable energy source i. i.e. lot_{mijt}. An example of the defined structure of the 3D chromosome is shown in Table 2 for $t=1$.

Table 2 The structure of the 3D chromosome

t=1	*j*=1	*j*=2	*j*=3
i=1	192220	111570	0
i=2	159410	0	0
i=3	0	0	0
i=4	10710	0	4400

Step 2. The population is sorted based on the non-domination criterion. Each solution (chromosome) is assigned a rank equal to its non-domination level where 1 is the best level, 2 is the next-best level and so on.

Step 3. Crowding distance is determined to distinguish among the solutions of the same rank, using crowded-comparison operator ($\prec_n$) which leads the selection process at the various stages of the algorithm toward a uniform spread-out Pareto optimal front. When two solutions are located at the same front, the solution that is placed in a lesser crowded region is preferred.

Step 4. Binary tournament selection with genetic operators is used to create an offspring population. Number of parents to be selected for creating children is specified with respect to crossover and mutation rates, which are set at 0.75 and 0.25 of the population size, respectively. Tournament selection involves running several "tournaments" among some individuals that are chosen randomly from the population. The winner of each tournament (the one with the best fitness) is selected for crossover and mutation. Number of parents selected for crossover is equal to *nc* that is $round(^{0.75\times nPop}/_{2})\times 2$. Since the variables are continuous, uniform crossover is used to create new chromosomes. To apply this type of crossover, a binary chromosome with the size equal to the size of the main chromosome is randomly generated, called α. Then, two new offsprings are created as follows:

Offspring 1 = $\alpha(parent1)+(1-\alpha)(parent2)$

Offspring 2 = $\alpha(parent2)+(1-\alpha)(parent1)$

Number of parents selected for crossover is equal to nm that is $round(0.25\times nPop)$. Flip mutation is applied in order to create offspring. In this type of mutation, the position of each row is substituted, i.e. the last row becomes the first one, the penultimate row becomes the second row and so on.

Step 5. Parent and offspring populations are mixed to make a new population with size 2×nPop. The new population is sorted by using non-domination criterion. Then, the new

[Downloaded from ijaor.ir on 2026-11-28]

population with size *nPop* is selected from the mixed population through the application of elitism and crowding distance, as the next generation.

Step 6. All the chromosomes with non-domination level 1 are sent into the archive and all dominated and duplicated solutions in the archive will be eliminated.

Step 7. Stopping criterion which is the maximum number of iterations (maxit=20) is checked. If it is not satisfied, we would return to *step 4*, otherwise, the archive in the final iteration is the candidate Pareto optimal set.

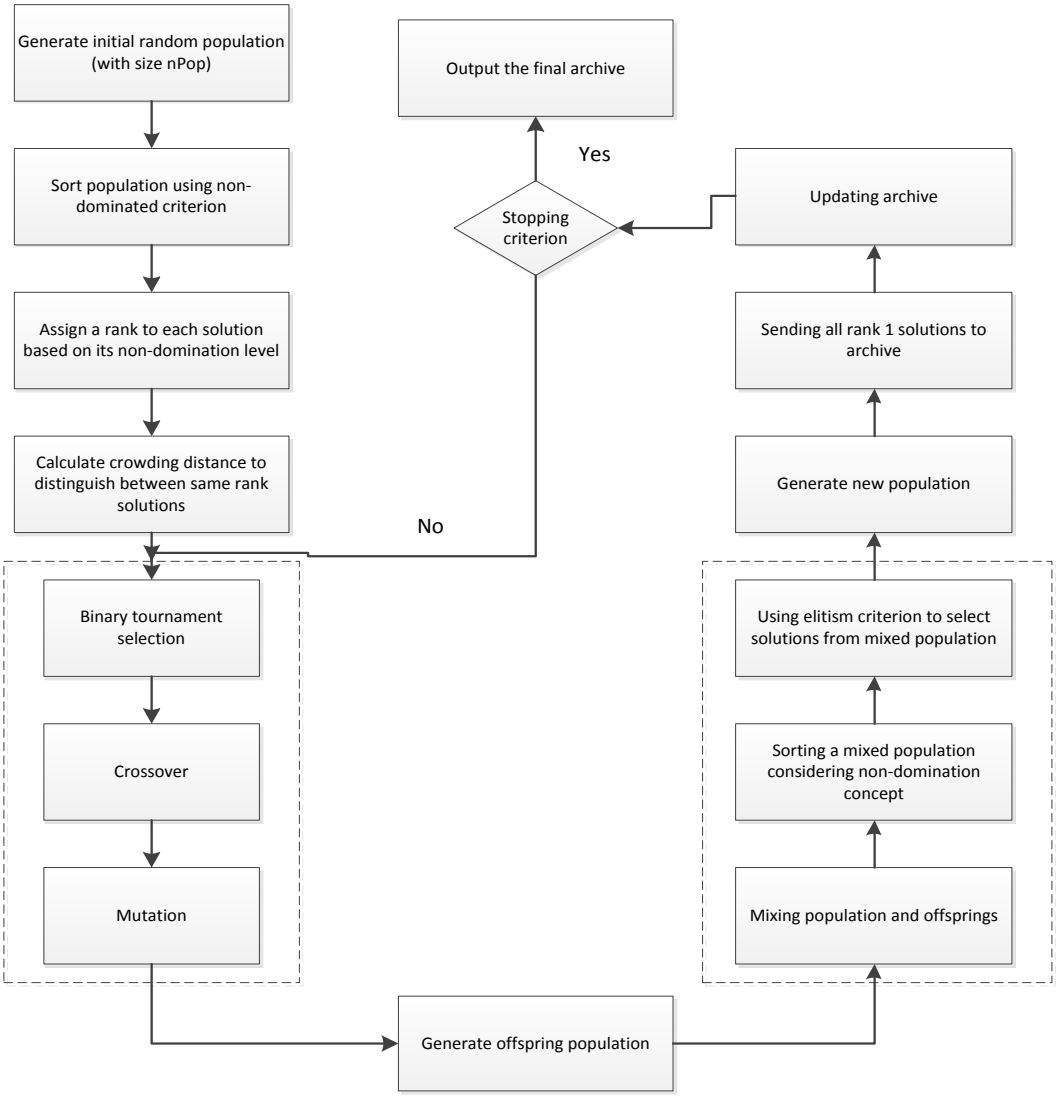

Fig. 2 Flowchart of the NSGA-II

5 Numerical results

In order to show the applicability of the proposed multi-objective mathematical model, numerical examples are conducted based on the data extracted from the literature. The obtained results are discussed in three different cases for different sizes of the model. The values of the parameters are given in Table 3 and Table 4.

[Downloaded from ijaor.ir on 2026-11-28]

Table 3 The first series of parameters' values

	i=1 Geo	*i*=2 Hydro	*i*=3 PV	*i*=4 Wind
P_{ijt} [a]	8724000000	5872000000	8366000000	4426000000
$O_{ijt}+ M_{ijt}$ [a]	200000000	28260000	55500000	79100000
C_{ijt} [b]	0.03	0.039	0.398	0.02
Bve_{ijt} [b]	1000	1000	1000	1000
Pe_{ijt} [b]	1200	1200	1200	1200
A_i [c]	8000000	327000	555000	100000
eta_{ij} [c]	0.13	0.39	0.9	0.15
GHG_i [c]	90	25	41	170
EG_i [c]	0.27549	0.27549	1.466	0.4

[a] www.eia.gov; [b] Kazemi & Rabbani [15]; [c] These values have been assumed by the authors.

Table 4 The second series of parameters' values

	j=1 Urban	*j*=2 Rural	*j*=3 Remote
D_j [a]	277082	278915	5276
I_j [b]	12	8	5

[a] Kazemi & Rabbani [15]; [b] These values have been assumed by the authors

5.1 Case 1: Small scaled problem ($T = 5$)

To solve the small size of the model with GAMS 22.9 software, four objective functions are combined by using the weighted sum method. Table 5 shows the optimal values through solving four single objective models in which just one objective is considered. Optimal values obtained from solving the WSM model are shown in Tables 5 and 6.

Table 5 Optimal values of single objective functions

Function	Value	Weight
f_1^+	1.000E+10	0.25
f_2^+	2.7090E+7	0.25
f_3^+	8.0205E+6	0.25
f_4^+	1.033E-14	0.25

Table 6 Optimal values for WSM model

Obtained values by WSM model			
X_{111}	1.3589E+5	Lot_{2111}	8.0147E-6
X_{121}	1.0811E+5	Lot_{2113}	8.0147E-6
X_{131}	1.3455E+5	Lot_{2114}	8.0147E-6
X_{211}	70594.857	Lot_{2115}	8.0147E-6
X_{221}	72377.464	Lot_{2121}	8.0147E-6
X_{231}	1.3455E+5	Lot_{2123}	8.0147E-6
X_{311}	70594.857	Lot_{2124}	8.0147E-6
X_{321}	72377.464	Lot_{2125}	8.0147E-6
X_{331}	1.3455E+5	Lot_{2131}	8.0147E-6

[Downloaded from ijaor.ir on 2026-11-28]

X_{413}	7.2432E-8	Lot_{2133}	8.0147E-6
X_{414}	7.2432E-8	Lot_{2134}	8.0147E-6
X_{415}	7.2432E-8	Lot_{2135}	8.0147E-6
X_{421}	26046.143	Lot_{2211}	8.0147E-6
X_{423}	7.2432E-8	Lot_{2213}	8.0147E-6
X_{424}	7.2432E-8	Lot_{2214}	8.0147E-6
X_{425}	7.2432E-8	Lot_{2215}	8.0147E-6
X_{431}	1.2395E+5	Lot_{2221}	8.0147E-6
X_{433}	3.6216E-8	Lot_{2223}	8.0147E-6
X_{434}	3.6216E-8	Lot_{2224}	8.0147E-6
X_{435}	3.6216E-8	Lot_{2225}	8.0147E-6

Figure 3 shows that how much each energy source has contributed in electricity generation for areas under study. The obtained results determine PV as the best renewable energy and bonds as the best capital resource in the portfolio for electrifying projects.

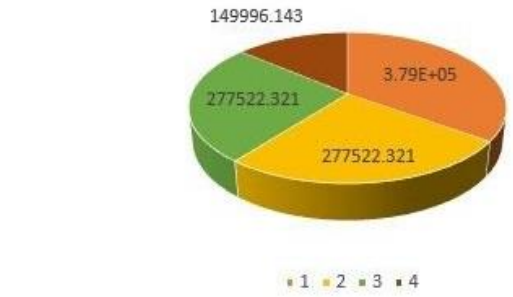

Fig. 3 Role of each energy source in electricity generation

5.2 Case 2: Medium scaled problem ($T = 10$)

Medium scale problem is solved by NSGA-II. The Pareto optimal solutions obtained in five different runs of the algorithm are given in Table 7. The value of objective functions shown in the Table is a solution that is selected randomly from all the Pareto optimal solutions obtained in each run.

Table 7. NSGA-II Pareto optimal solutions for Case 2

	Number of Pareto	Diversity	Spacing	Function 1	Function2	Function3	Function4	CPU time (s)
1	228	4.42e+015	1.5	1.085e+31	279276050.8	556015997.31	0.1812	28.98
2	215	5.17e+016	1.76	1.297e+30	287021210	539821529.08	0.1737	28.91
3	228	2.58e+015	1.62	3.15e+28	293850243.8	547705150.31	0.1581	29.74
4	221	3.72e+015	1.568	6.64e+29	302896429.7	572852684.39	0.1635	27.94
5	239	2.70e+015	1.67	2.75e+28	273718625	451967184.95	0.1385	24.42

Diversity and spacing specifies that how appropriate is a solution. More diversity shows that the algorithm has searched larger ranges of the decision space and the obtained solutions have more variety. Spacing shows the standard deviation of distances between Pareto solutions and the average distance. Hence, smaller spacing values help to find the Pareto frontier more exactly. Diversity (D) and spacing (S) can be calculated as follows:

[Downloaded from ijaor.ir on 2026-11-28]

$$D = \sqrt{\sum_{i=1}^{n} \max(\| x_t^i - y_t^i \|)} \tag{17}$$

$$S = \frac{\sum_{i=1}^{N-1} | \bar{d} - d_i |}{(N-1)\bar{d}} \tag{18}$$

Where $\| x_t^i - y_t^i \|$ is the Euclidean distance between two neighbor solutions x_t^i and y_t^i, d_i is the Euclidean distance between two sequential solutions on the optimal boundary, and $\bar{d}$ shows the average of solutions. The values of decision variables of each run for the last time period $(T = 10)$ are shown separately in Appendix. Obtained results show net present value of the selected projects is a large positive number, which indicates economic feasibility of projects. In addition, it seems that there are more tendencies to use bonds for funding renewable energy projects. PV is contributed in electricity generation more than other renewables, followed by hydro, wind, and geothermal.

5.3 Case 3: Large scale problem ($T = 25$)

Large scale problem is solved by NSGA-II. The Pareto solutions obtained in five different runs of the algorithm are given in Table 8. The value of objective functions shown in the Table is a solution that is selected randomly from all the Pareto optimal solutions obtained in each run.

Table 8. NSGA-II Pareto optimal solutions for Case 3

	Number of Pareto	Diversity	Spacing	Function1	Function2	Function3	Function 4	CPU time (s)
1	270	1.98e+016	1.743	1.50e+30	785917774	51678211588.08	0.1891	38.75
2	273	655935944907817	1.39	1.106e+31	785852727	51265502921.36	0.1915	38.91
3	260	514929705840701	1.29	1.75e+27	811943488	4956022514.58	0.1750	40.50
4	247	2.384e+016	1.66	2.38e+16	742346894.90	4901499663.99	0.1449	37.04
5	260	5.234e+015	1.64	1.15e+27	746500735.08	5270010974.43	0.1631	38.14

The values of decision variables of each run for the last time period **($T = 25$)** are shown separately in Appendix. The results show a high degree of feasibility for renewable energy projects because the net present value of the selected projects is a large positive number. Bonds are used for funding projects more than common stocks and bank loans. Also, PV is the best energy source to invest in electrifying projects.

[Downloaded from ijaor.ir on 2026-11-28]

5 Conclusion

In this study, the problem of investment in renewable energy sources for electricity generation was investigated. It was assumed that bonds, common stocks, and bank loans are three financial sources that can be used to fund renewable energy projects. The optimal portfolio for investment should be determined which minimizes weighted capital cost of investors. A multi-objective mixed integer mathematical model was developed in a way that four objectives, including maximization of NPV, minimization of GHG, maximization of employment generation, and minimization of the weighted cost of capital, were taken into consideration. A weighted sum method implemented in GAMS software was applied in order to integrate four objective functions into a single. Because of the NP-hard nature of the proposed model, GAMS software is unable to solve large scale problems. The large scales of the problem were solved by NSGA-II, which is a well-known and appropriate meta-heuristic solution approach for multi-objective optimization problems. Obtained results from computational experiments indicated that photovoltaic energy outperforms other renewable resources for electricity generation, especially in urban and rural areas. Also, bonds are the most attractive capital sources that possess larger shares of the portfolio for investment in renewables. According to the data utilized in this paper, despite of the high initial investment cost needed for establishing photovoltaic energy power plant as well as almost high operational and maintenance cost, photovoltaic provides the best combination of values for objective functions when we consider the same importance for each objective function. Given the results obtained in this paper, it would be rational to apply this type of renewable energy, if we had efficient financial sources for investment. But, in real cases always limitations in financial issues are available. Moreover, the results indicate that bonds have the higher portion in the construction of different power plants with respect to this set of data. This conclusion can be predictable because of the less average value considered for cost of capital of loans in different problems.

For future study, the problem studied here can be investigated under more complex situations such as considering other financial resources, incorporating inflation rate, applying other economic techniques for determining feasibility of the projects, considering restrictions due to different forms of dependency among the projects.

References

1. Varun R, Prakash IK, Bhat. 2009. Energy, Economics and environmental impacts of renewable energy systems, Renewable and Sustainable Energy Reviews 13: 2716-2721.
2. Rabbani, M., Heidari, R., Farrokhi-Asl, H., & Rahimi, N. (2018). Using metaheuristic algorithms to solve a multi-objective industrial hazardous waste location-routing problem considering incompatible waste types. Journal of Cleaner Production, 170, 227-241.
3. Bektaş, T., & Laporte, G. (2011). The pollution-routing problem. Transportation Research Part B: Methodological, 45(8), 1232-1250.
4. Bergmann A, Colombo S, Hanley N. 2008. Rural versus urban preferences for renewable energy developments, Ecological Economics: 616-625.
5. Pantaleo A, Pellerano A, 2005. Ruggiero F, Trovato M. Feasibility study of off-shore wind farms: an application to Puglia region, Solar Energy 79: 321–331.
6. Ozerdem B, Ozer S, Tosun M. 2006. Feasibility study of wind farms: A case study for Izmir, Turkey, Journal of Wind Engineering and Industrial Aerodynamics 94: 725–743.
7. Kahraman C, Kaya I, Cebi S. 2009. A comparative analysis for multi attribute selection among renewable energy alternatives using fuzzy axiomatic design and fuzzy analytic hierarchy process, Energy 34: 1603-1616.

[Downloaded from ijaor.ir on 2026-11-28]

8. Akdag SA, Guler O. 2010. Evaluation of wind energy investment interest and electricity generation cost analysis for Turkey, Applied Energy: 2574-2580.
9. Yang, J., Chen, W., Chen, B., & Jia, Y. (2012). Economic feasibility analysis of a renewable energy project in the rural China. Procedia Environmental Sciences, 13, 2280-2283.
10. Trapani, K., Millar, D. L., & Smith, H. C. (2013). Novel offshore application of photovoltaics in comparison to conventional marine renewable energy technologies. Renewable energy, 50, 879-888.
11. Hashim, H., Douglas, P., Elkamel, A., & Croiset, E. (2005). Optimization model for energy planning with CO2 emission considerations. Industrial & engineering chemistry research, 44(4), 879-890.
12. Li, Y. P., Huang, G. H., & Chen, X. (2011). Planning regional energy system in association with greenhouse gas mitigation under uncertainty. Applied Energy, 88(3), 599-611.
13. Frondel, M., Ritter, N., Schmidt, C. M., & Vance, C. (2010). Economic impacts from the promotion of renewable energy technologies: The German experience. Energy Policy, 38(8), 4048-4056.
14. Kazemi, S. M., & Rabbani, M. (2013). An integrated decentralized energy planning model considering demand-side management and environmental measures. Journal of Energy, 2013.
15. Masini, A., & Menichetti, E. (2012). The impact of behavioural factors in the renewable energy investment decision making process: Conceptual framework and empirical findings. Energy Policy, 40, 28-38.
16. Herran, D. S., & Nakata, T. (2012). Design of decentralized energy systems for rural electrification in developing countries considering regional disparity. Applied Energy, 91(1), 130-145.
17. Hiremath, R. B., Shikha, S., & Ravindranath, N. H. (2007). Decentralized energy planning; modeling and application—a review. Renewable and Sustainable Energy Reviews, 11(5), 729-752.
18. Pelet, X., Favrat, D., & Leyland, G. (2005). Multiobjective optimisation of integrated energy systems for remote communities considering economics and CO2 emissions. International Journal of Thermal Sciences, 44(12), 1180-1189.
19. Bernal-Agustín, J. L., Dufo-López, R., & Rivas-Ascaso, D. M. (2006). Design of isolated hybrid systems minimizing costs and pollutant emissions. Renewable Energy, 31(14), 2227-2244.
20. Akella, A. K., Saini, R. P., & Sharma, M. P. (2009). Social, economical and environmental impacts of renewable energy systems. Renewable Energy, 34(2), 390-396.
21. Katsigiannis, Y. A., Georgilakis, P. S., & Karapidakis, E. S. (2010). Multiobjective genetic algorithm solution to the optimum economic and environmental performance problem of small autonomous hybrid power systems with renewables. IET Renewable Power Generation, 4(5), 404-419.
22. San Cristóbal, J. R. (2012). A goal programming model for the optimal mix and location of renewable energy plants in the north of Spain. Renewable and Sustainable Energy Reviews, 16(7), 4461-4464.
23. Rabbani, M., Ramezankhani, M. J., Farrokhi-Asl, H., & Farshbaf-Geranmayeh, A. (2015). Vehicle routing with time windows and customer selection for perishable goods. International Journal of Supply and Operations Management, 2(2), 700.
24. Rabbani, M., Montazeri, M., Farrokhi-Asl, H., & Rafiei, H. (2016). A multi-objective genetic algorithm for a mixed-model assembly U-line balancing type-I problem considering human-related issues, training, and learning. Journal of Industrial Engineering International, 12(4), 485-497.
25. Talbi, E. G. (2009). Metaheuristics for multiobjective optimization. Metaheuristics: from design to implementation, 308-384.
26. Konak, A., Coit, D. W., & Smith, A. E. (2006). Multi-objective optimization using genetic algorithms: A tutorial. Reliability Engineering & System Safety, 91(9), 992-1007.
27. Holland, J. H. (1992). Adaptation in natural and artificial systems: an introductory analysis with applications to biology, control, and artificial intelligence. MIT press.
28. Deb, K., Pratap, A., Agarwal, S., & Meyarivan, T. A. M. T. (2002). A fast and elitist multiobjective genetic algorithm: NSGA-II. IEEE transactions on evolutionary computation, 6(2), 182-197.
29. Rabbani, M., Farrokhi-Asl, H., & Asgarian, B. (2017). Solving a bi-objective location routing problem by a NSGA-II combined with clustering approach: application in waste collection problem. Journal of Industrial Engineering International, 13(1), 13-27.

[Downloaded from ijaor.ir on 2026-11-28]

Appendix

In this section, values of the decision variables of each run for the last time period in medium and large size problem have been reported in Tables A.1-A.4.

Table A.1. Values of x_{ijt} for Case 2 at $t = 10$

		X_{ijt}		
Run 1	i	j		
		7022.4	8716.09	0
		127530	0	5275.2
		127530	255199	0
		15000	15000	0
Run 2	i	j		
		7513.6	14149.8	4719.04
		127530	1003.6	2222.3
		127038.28	497520	0
		15000	15000	0
Run 3	i	j		
		9112.4	12376.8	0
		127530	116211.7	0
		127530	113787	1534.8
		6399	15000	0
Run 4	i	j		
		0	0	0
		124626	127529.75	0
		133602.7	151384.4	0
		15000	0	5275.2
Run 5	i	j		
		0	136385	0
		127530	127529.9	0
		134552.04	0	5275.8

[Downloaded from ijaor.ir on 2026-11-28]

		15000	15000	0

Table A.2. Values of lot_{mijt} for Case 2 at $t = 10$

		Lot_{ij1t}			Lot_{ij2t}			Lot_{ij3t}		
Run 1	*i*	*j*			*j*			*j*		
		148987067507.7	16552832538.9	13604025962.3	0	0	272109528879.06	0	0	0
		342282.8	133734.28	1616293.5	0	0	0	765089649.08	0	3660793.31
		76130854.2	43737225.95	1481476.7	0	0	0	0	0	0
		132889492.12	7669394333.14	332242206.6	0	0	0	279904693	272362664.9	77582199.68
Run 2	*i*	*j*			*j*			*j*		
		1159878183.1	14358187672.3	7039167728	398814396.2	3758235657	1021812530.6	57946988702	46450095246.7	18846187294
		38204135709	3511713975.4	5126778719.0	9891001538.6	7786233655	41710324501	61201638.87	82469787	188464203
		2940060850	3226279308.3	308742457092	69046523735434	9336073936670	20785517454028	683194319.1	19993291693	2121190197
		3298190527.4	8761116145.04	551546522.7	40358243865.6	6092343019.3	39874054692.8	34920177.4	5630386798	8675957388
Run 3		130576737.3	306565213	144578872.9	55726431964.2	53131160020.7	10850822490.5	0	0	0
	i	*j*			*j*			*j*		
		23059369	430168.9	259437.3	69911623831.0	26924292860.3	1578180810.2	0	0	0
		12527310124.3	29414394131.5	10666335863.6	1230697196681.8	233114696409	161681316999	2270901213.6	2100128995.4	101314956.8
		3015618130.2	47244.05	47959.2	52947863124.5	88403422361.5	180771288829	0	0	0
Run 4	*i*	*j*			*j*			*j*		
		10238605.5	29166394.5	285822276.7	0	0	0	1567089819.8	2042851544.5	841004105.6
		1734177246.3	48749444371.5	4127246910	0	0	2372433176	0	0	0

[Downloaded from ijaor.ir on 2026-11-28]

		6126749 0730	804019 18217.8	636556 95246	0	0	0	0	0	0
		0	0	0	0	0	0	0	1963330 54530.2	0
Run 5	*i*	*j*			*j*			*j*		
		5507497 413.4	411742 6806	287901 75368.4	1974683 5538.3	3540294 0355.5	0	1422293 12262.7	9562991 904.8	1006329 61420.1
		0	0	0	2095821 1627.3	8044782 693	1713181 8797.4	1688445 7115.7	3201965 2.5	9539935 332.8
		0	0	0	0	0	0	2334706 35	14872.6	8247680 4.3
		0	346730 11836.7	524018 88575	0	0	0	0	0	0

Table A.3. Values of x_{ijt} for Case 3 at $t = 25$

		X_{ijt}		
Run 1	*i*	*j*		
		15000	15000	0
		0	125474.2	0
		250685.7	127530	5275.75
		250685.7	10910.8	0
Run 2	*i*	*j*		
		134551	0	5276
		127530	0	0
		0	278914.9	0
		15000	0	0
Run 3	*i*	*j*		
		134551	278913	5276
		127528.5	0	0
		134551.8	0	0
		15000	0	0

[Downloaded from ijaor.ir on 2026-11-28]

Run 4	i	j		
		11454	14303.2	0
		0	127530	0
		252892	137081	0
		12645	0	5275.8
Run 5	i	j		
		16323.29	24375	0
		127539	127530	0
		127530	127530	5276
		0	0	0

Table A.4. Values of lot_{mijt} for Case 3 at $t = 25$

		Lot_{ij1t}			Lot_{ij2t}			Lot_{ij3t}		
Run 1	i	j			j			j		
		469546167.6	4563267797.6	129591805.2	280696091559.9	74826848992.7	0	43111072.3	13775645 82.6	41017527.6
		1511149.5	1897533.5	11393365.2	9370993227.3	26310572.5	0	0	0	0
		66215910746	7831689280.4	16303946181.5	74319589562.6	25929366 21.6	0	0	0	0
		114204188730.3	9440957243	0	812155620.1	903017977.8	0	0	10340681 7.4	0
Run 2	i	j			j			j		
		115534042233.68	19575515 9573.2	43214028270.5	0	0	0	59164329 77.1	22068138898.3	39849216503.2
		0	0	0	0	0	0	0	0	0
		0	0	0	212770300.3	127263561	69526603 2.4	0	0	0
		32440069922.66	0	0	0	0	0	0	0	0
Ru	i	j			j			j		

[Downloaded from ijaor.ir on 2026-11-28]

n 3		0	0	0	1364154 73618.8	1988156 42918.3	3480415 7614.6	8277098 898.8	1556152 39719.9	1058766 1381.1
		17578665 2540.4	3193196 8643	3768438 8533.9	0	0	0	0	0	0
		0	0	0	0	0	0	0	0	0
		20959874 161.5	1054445 86237.3	1781699 6934.9	0	0	0	0	0	0
R u n 4	*i*	*j*			*j*			*j*		
		78882866 883.7	0	0	0	0	0	1558710 509.3	8237876 20.7	2251289 0170.4
		0	0	0	0	0	0	0	0	0
		26712780 87	2320681 9.9	2155747 72.2	5841997 6.6	0	6754932 62	0	0	0
		0	5131219 697.5	0	0	0	2596310 430.5	4455661 1984.7	1815279 99715	0
R u n 5	*i*	*j*			*j*			*j*		
		25266017 3.5	7161655 97.6	1646180 193.8	1351325 54.4	3613332 64.3	4641733 359.4	2335301 26	3886048. 5	3223824 .6
		23305601 053.9	3411605 0313.9	7952085 208.4	3304163 8471	6630807 8098.3	6008883 5426.4	0	0	0
		20373262 417.9	2365342 19965.5	6166491 2.5	1335894 4.9	4186.8	1050507 73.2	0	0	0
		3309015. 4	1981147. 4	0	0	0	6325349 4921.3	2974421 4.52	2067023 13.08	5571347 2

[Downloaded from ijaor.ir on 2026-11-28]