International Journal of Applied Operational Research
Vol. 11, No. 1, pp. 33-45, Winter 2023

Journal homepage: ijorlu.liau.ac.ir

Determining a common set of weight by reducing the flexibility of weight profile

A. Pourhabib*, M. Maghbouli

Received: 23 July 2021 ; **Accepted:** 10 February 2022

Abstract Data Envelopment Analysis (DEA) is non-parametric mathematical programming for measuring the performance of a set of homogeneous decision-making units (DMUs). Standard DEA models usually result in several efficient units, so, picking the best unit among efficient units has been one of the most challenging subjects in DEA literature. With reference to various researchers, the common set of weights (CSW) approach has been intriguing among them. This paper discusses a mechanism for detecting a common set of weights which is managed to be always positive and prevents weights dissimilarity. Employing this common set of weights can determine the efficiency score of each unit and finally rank them based on their obtained efficiency score. Equally, the proposed model not only provides the closest targets, but also minimizes the deviations of actual DMUs and extreme efficient units. In order to verify the proposed approach an empirical example of Iranian electricity distribution companies is explained.

Keywords: Data Envelopment Analysis (DEA), Common Set of Weight (CSW), Efficiency Score, Deviation and Weight dissimilarity.

1 Introduction

Data envelopment analysis (DEA) is concerned with a comparative assessment for evaluating the efficiency of decision-making units (DMU). One of the most challenging issues in DEA literature is the concept of input /output weights. Standard DEA models produce more than one efficient unit and determine the weights of inputs and outputs separately for each DMU. So, the flexibility in choosing weights has been questioned. From the practical point of view, picking one or more efficient units looks imprecise and impossible. Consequently, the different sets of weights can lead to different efficiency measures for DMUs. So, the flexibility in selecting input/output weight applying different sets of weights can be argued. This means that employing a common set of weights can reduce flexibility. To overcome a common set of weights (CSW) problem, many researchers have been established and extended in DEA literature. For instance, Ganley,

* **Corresponding Author. (✉)**
E-mail: apourhabibyekta@yahoo.com (A. Pourhabib)

Azam Pourhabib
Department of Applied Mathematics, Sowmesara Branch, Islamic Azad University, Sowmesara, Iran

Mahnaz Maghbouli
Department of Applied Mathematics, Aras Branch, Islamic Azad University, Jolfa, Iran

[Downloaded from ijaor.ir on 2026-11-28]

[DOI: 10.71885/ijorlu-2023-1-619]

and Cubbin [1] determined the common set of weights by maximizing the summation of unit performance. Roll, Cook, and Golany [2] and Roll and Golany [3] proposed several approximations for common weights production. The authors implemented unbounded DEA models for generating various sets of weights. By taking a weighted average on efficiency scores as the weights, the proposed models maximize the average efficiency of units and the number of efficient DMUs. Although, according to order of importance, various factors can be ranked by their model. As the last step, low weights can be assigned to less important factors and a maximal feasible weight goes to important ones. Sinuany- Stern et al. [4] suggested a two stage linear discriminate analysis approach to produce the common weights. Sinuany-Stern, and Friedman [5] argued a nonlinear discriminate analysis to provide the common weights. Kao and Hung [6] presented the comparison solution approach to generate a common set of weights under the DEA framework. The idea behind the model is searching for common set of weights achieving the shortest distance between the efficiency score calculated from the corresponding weights and the targets. This target is the efficiency score calculated from the standard DEA model. Liu and Peng [7] just focused on efficient DMUs and proposed an approach to identify a common set of weights for the performance indices. Wang et al. [8] introduced a methodology by imposing an appropriate minimum weight restriction on all inputs and outputs that rank all DMUs. In another attempt, Wang et al [9] proposed an alternative method based on regression analysis to search a common set of weights for fully ranking DMUs. Sun et al. [10] suggested two different models with reference to ideal and anti- ideal DMUs to conduct common weight for efficiency scores then ranking units. Surveying these researches, this paper proposes a model for determining a common set of weights which evaluates the absolute efficiency of each unit. The contribution of the paper is three folded. First, the proposed method provides the closest targets on the efficient frontier for each input and output. Especially, the targets on the efficient frontier satisfy the update characteristics and composed of extreme efficient units at the same time. Second, the proposed model allows minimizing the deviations of actual inputs and outputs with the determined target. Third, the model generates positive input/output weights and prevents weights' dissimilarity simultaneously. As a non- parametric technique, the proposed model does not require the initial information on input /output's weights. Top of all, in efficiency evaluation and production estimate, the results of the proposed model is more trust able.

The structure of this paper unfolds as follows. The following section is briefly speaking about traditional DEA models and discusses some necessary properties. Section 3 extends our proposed methodology for determining a common set of weights. In Section 4, a real -world example of electricity distribution companies in Iran is analyzed to illustrate the applicability of the proposed approach. The Conclusion will end the paper.

2 Preliminaries

Consider that there are n production units (DMU) that can be evaluated in terms of m inputs and s outputs. Let $x_{ij}(i=1,...,m)$ and $\mathbf{y}_{rj}$ $(r=1,...,s)$ be the input and output vectors of DMU_j $(j=1,...,n)$. Also, imagine u_r $(r=1,...,s))$ is the weight vector given to r-th output and v_i $(i=1,...,m)$ plays the role of the weight vector given to i-th input. According to Charnes et.al

[Downloaded from ijaor.ir on 2026-11-28]

[DOI: 10.71885/ijorlu-2023-1-619]

[11], the best relative efficiency of each unit can be measured by following CCR* model that was named by the acronym of the three authors:

$$Max \quad Eff_o = \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}}$$

$$s.t. \quad \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0, \quad j = 1,...,n, \qquad (1)$$

$$u_r, v_i \ge 0, \quad for\ all\ r,\ i.$$

In the above model, $DMU_o (o = 1,...,n)$ refers to the DMU under evaluation. If the optimal value of the objective function for DMU_o equals to unity, then the under evaluated unit is efficient. Otherwise, it is called as inefficient. For more description, consider the dual format of Model (1). The dual formulation of Model (1) can be stated as follows.

$$Min \quad \theta_o$$

$$s.t. \quad \sum_{j=1}^{n} \lambda_j x_{ij} \le \theta_o x_{io}, \ i = 1,...,m,$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{ro} \ , \ r = 1,...,s \qquad (2)$$

$$\lambda_j \ge 0, \quad j = 1,...,n.$$

In the model above, θ_o indicates the efficiency score of the under evaluated unit. Also, $\lambda_j (j = 1,...,n)$ shows the intensity variable of each unit. The standard CCR model, model (1), applies the unit invariant property. This property has been structured by Lovell and Pastor [12] which employs to normalize the weights. Since the main interest of our study is to prevent weight dispersion; this valuable property can be implemented on model (1) as constraints (3-2) and (3-3) admits. In other words, there is a scale of data leading to the equivalent form of model (1) as follows:

$$Max \quad Eff_o = \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}} \qquad (3)$$

$$s.t.$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0, \quad j = 1,...,n, \quad (3-1)$$

$$0 \le u_r \le 1, \quad r = 1,...,s \qquad (3\text{-}2)$$

$$0 \le v_i \le 1, \quad i = 1,...,m \qquad (3\text{-}3)$$

* Charnes, Cooper and Rhodes (CCR)

[Downloaded from ijaor.ir on 2026-11-28]

[DOI: 10.71885/ijorlu-2023-1-619]

Suppose that $u_r^*\ (r=1,...,s)$ and $v_i^*\ (i=1,...,m)$ are optimal solutions of model (1) and φ defined as the maximum of optimal weight values of $u_r^*\ (r=1,...,s)$ and $v_i^*\ (i=1,...,m)$. If the weights of input and output of the model (1) are divided to φ, an answer for model (3) is assessed. So, models (1) and (3) are equivalent. As it can be seen the objective function and first constraint of models (1) and (3) are similar. But nonnegative variables in the model (1) have been replaced with the bounded variables in the model (3). The bounded variables in the model (3) can prevent dissimilar weights. This property is the advantage of the model (3) in contrast to the model (1). In the following section, a developed model is proposed which can support the generation of nonnegative weights. Model (3) has been taken from the article by Pourhabib et al [13].

3 The Proposed Approach

Once we obtain the optimal solution of model (1), each unit selects its best weights to maximize the efficiency score. However, some questions are raised. First, different sets of weights may result in different efficiency scores. Hence, comparing the scores and ranking the units is disputable. Secondly, the standard DEA models always generate more than one efficient unit which leads to a lack of discrimination among units. In order to tackle with these shortcomings, a common set of weight (CSW) approach has been proposed to reduce the flexibility in weight selection. Based on the idea behind the model (3), an alternative CSW mechanism is developed in this section. Again suppose that there are n production units $DMU_j\ (j=1,...,n)$ that consume varying amount of m inputs $x_{ij}\ (i=1,...,m)$ to produce s outputs $y_{rj}\ (r=1,...,s)$. The production possibility set (PPS) T can be described as:

$$T=\left\{(x,y), x\geq\sum_{j=1}^{n}\lambda_j x_j, y\leq\sum_{j=1}^{n}\lambda_j y_j, \lambda_j\geq 0\right\}.$$

This set has set up on the constant return to scale (CRS) assumption for the production technology and employs to evaluate the efficiency of all $DMUs$. As a matter of fact, each unit should catch its target on the efficient frontier which formed by some extreme efficient $DMUs$. However, the aim is to set the closest targets which can be achieved, especially for inefficient $DMUs$. In doing so, the proposed model employs a mechanism that can provide the closest target after the linear combinations of extreme units are allowed. So, the proposed model seeks to minimize the deviations of actual inputs and outputs from the given targets. Model (4) is then formulated as follows:

[Downloaded from ijaor.ir on 2026-11-28]

[DOI: 10.71885/ijorlu-2023-1-619]

$$Eff^{new} = Min \quad \frac{\sum_{j=1}^{n}\left[\sum_{i=1}^{m}\left(x_{ij} - xh(i)\right) + \sum_{r=1}^{s}\left(y_{rj} - yh(r)\right)\right]}{\varphi}$$

s.t.

$$xh(i) = \sum_{\substack{j=1 \\ j\in E}}^{n} \lambda_j x_{ij}, i = 1,\ldots,m \qquad (4-1)$$

$$yh(r) = \sum_{\substack{j=1 \\ j\in E}}^{n} \lambda_j y_{rj}, r = 1,\ldots,s, \qquad (4-2)$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \; j = 1,\ldots,n, \qquad (4-3)$$

$$\sum_{r=1}^{s} u_r yh(r) - \sum_{i=1}^{m} v_i xh(r) = 0, \qquad (4-4)$$

$$\varphi \le v_i \le 1, \quad i = 1,\ldots,m, \qquad (4-5)$$

$$\varphi \le u_r \le 1, \quad r = 1,\ldots,s, \qquad (4-6)$$

$$u_r, v_i, \varphi, \lambda_j \ge 0 \text{ for all } i, j \text{ and } r, \qquad (4-7)$$

Clearly, Model (4) is a nonlinear program and focuses on two main features. According to constraints (4-5) and (4-6) the model prevents weight dissimilarity. These constraints screw all weights to settle between two bounds: the common lower bound φ and the upper bound of unity. In this way, it prevents the dispersion of input and output weights. The second feature of the model (4) is determining positive common set of weights. The model (4) assumes that all observation belong to the production frontier as indicated by the constraint $\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0$, $j = 1,\ldots,n$. Also, this constraint can define all hyper planes of the efficient frontier. The first two constraints (4-1) and (4-2) represent the linear combinations of extremely efficient units. In fact, the presence of constraint (4-4) attempts to catch such a set of weights conducted by extreme efficient units. This model attempts to pick up a set of weights conducted by extremely efficient units. This strength of the model leads to set positive and dissimilar weights. In addition, guarantees that we can select a common set of weights among the multipliers of supporting hyper planes. The objective function of the model (4) can support the idea of minimum deviation. Looking closely, deviation of each unit and the virtual one is minimized. As model (4) admits, the virtual unit is composed of linear combination of extreme efficient units. Note that, setting this minimum deviation in the objective function, the lower bound φ, can be increased. What's more, by proper choice for φ , among all feasible multipliers, our proposed model can effectively avoid weight dissimilarity and generates positive common weight and at the same time. Notably, this process in common set of weight choice does not require additional information about the unit under evaluation. The initial information about inputs and outputs is sufficient. Likewise, this process has an influence on optimal solutions. That is to say: the common set of weights can be

[Downloaded from ijaor.ir on 2026-11-28]

[DOI: 10.71885/ijorlu-2023-1-619]

selected by jointly restricting the input and output weights with a single bound. However, this common lower bound of input/output weights, φ, may not be sufficient to achieve optimal weights. Hence, this model can be extended to a general model that can restrict input and output weights separately. To address this issue, the following linear fractional programming problem is proposed.

$$Min \quad \frac{\sum_{j=1}^{n}\left[\sum_{i=1}^{m}\left(x_{ij}-xh(i)\right)+\sum_{r=1}^{s}\left(y_{rj}-yh(r)\right)\right]}{\varphi}$$

s.t.

$$xh(i)=\sum_{\substack{j=1\\ j\in E}}^{n}\lambda_j x_{ij}, i=1,\ldots,m \qquad (5-1)$$

$$yh(r)=\sum_{\substack{j=1\\ j\in E}}^{n}\lambda_j y_{rj}, r=1,\ldots,s, \qquad (5-2)$$

$$\sum_{r=1}^{s}u_r y_{rj}-\sum_{i=1}^{m}v_i x_{ij}\le 0\ j=1,\ldots,n, \qquad (5-3)$$

$$\sum_{r=1}^{s}u_r yh(r)-\sum_{i=1}^{m}v_i xh(r)=0, \qquad (5-4)$$

$$Z_I\le v_i\le 1, i=1,\ldots,m, \qquad (5-5)$$

$$Z_O\le u_r\le 1, r=1,\ldots,s, \qquad (5-6)$$

$$Z_I\ge\varphi, \qquad (5-7)$$

$$Z_O\ge\varphi, \qquad (5-8)$$

$$u_r, v_i, \varphi, Z_I, Z_O\lambda_j\ge 0 \text{ for all } i, j \text{ and } r,$$

The idea behind the model (5) is as same as that one in the model (4). But there exists a great divide between model (4) and model (5). The constraint (5-5) forces all input multipliers to screw between the lower bound Z_I and unity. The constraint (5-6) also makes all output multipliers lie down between the lower bound Z_O and the upper bound 1. The constraints (5-7) and (5-8) restrict the lower bound of input and output's weights to the positive variable φ. That is to say, input and output weights are restricted separately. The objective function of the model (5) is also established to maximize φ while the distance between Z_I and Z_O with their upper bounds is reduced. Thus, the model supplies the weights with the least dissimilarity, which is the main interest of the study. The following two theorems emphasize that both proposed models are always feasible. Besides, positive weights can be assessed by implementing these models.

Theorem 1: The proposed model (4) is feasible and generates positive weights in optimality.

Proof: Imagine DMU_o is evaluated by the CCR model (model (1)). Also, DMU_d plays the reference unit of DMU_o. Thus, we have $u_r^d>0$ and $v_i^d>0 (i=1,\cdots,m, r=1,\cdots,s)$. As a result,

[Downloaded from ijaor.ir on 2026-11-28]

[DOI: 10.71885/ijorlu-2023-1-619]

$u_r^d, v_i^d, \varphi = \underset{i,r}{Min}\left\{u_r^d, v_i^d\right\}$ is a feasible solution for evaluating DMU_o by the model (4). Also, note that that φ>0.

Theorem 2: The proposed model (5) is always feasible and in optimality φ>0.

Proof: the proof is the same as Theorem (1).

In order to highlight the models' applicability, two different examples are distinguished.

4 Numerical example

To verify the applicability of the proposed models, two various examples are executed. For the first try, the proposed model is compared with those proposed by Kao and Hung's [6] models. The authors have proposed three models employing different distance functions to generate common weights. For more information, refer to Kao and Hung [6].

Example 1

This example consists of 12 flexible manufacturing systems (FMSs) with two inputs and four outputs. These data are derived from Shang and Sueyoshi [14] and are recorded in Table 1. The inputs include annual operating and depreciation costs (x_1) and the floor space requirements of each specific system (x_2). Outputs signify the improvement of qualitative benefits (y_1), work in process (y_2), average number of tardy jobs (y_3), and average yield (y_4).

Table 1 Data set for 12 flexible manufacturing system (FMSs)

FMS	x_1	x_2	y_1	y_2	y_3	y_4
1	17.02	5	42	45.3	14.2	30.1
2	16.46	4.5	39	40.1	13	29.8
3	11.76	6	26	39.6	13.8	24.5
4	10.52	4	22	36	11.3	25
5	9.50	3.8	21	34.2	12	20.4
6	4.79	5.4	10	20.1	5	16.5
7	6.21	6.2	14	26.5	7	19.7
8	11.12	6	25	35.9	9	24.7
9	3.67	8	4	17.4	0.1	18.1
10	8.93	7	16	34.3	6.5	20.6
11	17.74	7.1	43	45.6	14	31.1
12	14.85	6.2	27	38.7	13.8	25.4

The proposed model (4) and Kao and Hung [6] models were implemented on this data set. Table 2 shows the results. The first three rows of Table 2 demonstrate three different results (based on different distance functions) and the last row shows the results of the proposed model (4). As Table (2) records the weights for output 3 in Kao and Hung [6] model are always zero, whilst the proposed model (4) gives strictly positive weights for all outputs. Note that the results of Kao and Hung [6] models are taken from Sun et al [10].

[Downloaded from ijaor.ir on 2026-11-28]

[DOI: 10.71885/ijorlu-2023-1-619]

Table 2 Common weights Derived by different models

Model	I_1	I_2	O_1	O_2	O_3	O_4
P=1	0.076680	0.026510	0.021330	0.004024	0	0.011944
P=2	0.928986	0.386831	0.225722	0	0	0.254175
P=∞	0.896384	0.415631	0.150772	0.05559	0	0.238215
Proposed model(4)	0.928542	1.00000	0.177336	0.191191	0.060618	0.127374

For detailed analysis, the efficiency scores of different models are depicted in Table 3. The second column is calculated by the traditional CCR model (1), the rest three columns present the efficiency score of Kao and Hung [6] models. The last column includes the efficiency scores of the proposed model (4).

Table 3 The efficiency scores calculated by different models

DMUs	CCR	P=1	P = 2	P = ∞	Eff^{new}
1	1	1	0.9654	0.9111	0.98990
2	1	0.9788	0.9616	0.9026	0.96880
3	0.9824	0.9488	0.9132	0.9021	0.95387
4	1	1	1	1	0.99998
5	1	1	0.9641	0.9663	1
6	1	0.9624	0.9866	0.9872	0.81451
7	1	1	1	1	0.87604
8	0.9614	0.9614	0.9423	0.9203	0.91813
9	1	0.7528	0.8462	0.8760	0.55643
10	0.956	0.8334	0.8041	0.8295	0.81175
11	0.9831	0.9507	0.9160	0.8591	0.89740
12	0.8012	0.7943	0.7750	0.7602	0.81340
Average	0.9734	0.9317	0.9228	0.9095	0.883351

The notification Eff^{new} indicates the performance score of the proposed model (4). The second column of Table 3 shows the CCR efficiency scores. The traditional CCR model (1) evaluates seven out of the twelve units as DEA efficient. This subject leads to a lack of discrimination power of the CCR model (1). The rest three columns of Table 3 derive the efficiency scores of Kao and Hung [6] models which are based on different distance functions. It can be seen, there are four efficient units in P=1and two efficient units in P = 2and P = ∞. As the last column of Table 3 presents, the proposed model (4) gives only one efficient unit. Therefore, the number of efficient units is reduced from 7 to 1 in the proposed model (4). From the statistical point of view, the average efficiency of all mentioned methods is listed in the last row of Table 3. The average of efficiencies in the proposed model (4) is 0.8833 while this quantity is recorded as 0.9734 in the CCR model (1) and 0.9317 in Kao and Hung [6] with p = 1. The other two models of Kao and Hung [12] with P = 2 and P = ∞, the average of efficiency scores are depicted as 0.9228 and 0.9095, respectively. As expected, the results show that the proposed model (4) outperforms the existing model. Notably, the proposed model (4) not only results in strictly positive weights compared to other models, but also avoids weight dissimilarity. Generally, model (4) reduces efficiency scores and the number of efficient units.

[Downloaded from ijaor.ir on 2026-11-28]

[DOI: 10.71885/ijorlu-2023-1-619]

Example 2

In this section, we apply our proposed models to analyze a real example of Iranian electricity distribution companies. Since electricity distribution has been pointed out in most of the researches, a summary of some studies has been given. Yuzhi and Zhangan [15] have studied the input-output efficiency of distribution systems from the more different aspects. Performance analysis of 21 Turkish electricity distribution companies was conducted by Celen [16]. Omrani et al. [17] employed a mixed methodology of bargaining game, principal component analysis, and DEA to evaluate the efficiency of electricity distribution in Iran. The ranking of the electricity distribution in Iran was carried out by Tavassoli et al. [18] with a view to strong complementary slackness conditions. All of these studies have focused on calculating the efficiency of electricity distribution companies. This section struggles to evaluate electricity distribution companies in Iran employing the proposed models. In this study, 39 electricity distribution companies in Iran with 14 variables including 6 inputs and 8 outputs are selected. Input variables include: transformer capacity or maximum amount of power that can be transformed by the transformer and denoted as (x_1), Number of Transformers in circuits is (x_2), Low voltage network or voltage levels less than 1 KV (x_3), Medium voltage network or voltage levels greater than 1kV and less 100 kV (x_4), Number of employees (x_5) and Area (x6). Output variables can be listed as the Energy delivery (y_1), Energy consumption of other customers or the total amount of energy used except industrial and household consumption (y_2), Industrial energy consumption (y_3), Household energy consumption (y_4), Number of other customers (y_5), number of industrial customers (y_6), Number of household customers (y_7), and Number of Lights of a street lighting (y_8). Table 4 represents the data set for these 39 companies.

Table 4 The data set for 39 electricity distribution companies

Company	x_1	x_2	x_3	x_4	x_5	x_6	y_1	y_2	y_3	y_4	$y_{5,7}$	y_6	y_8
Tabriz	1686	5590	5317	2966	572	4770	3816	1066	1154	1300	864	9	189
Azarbayejansharghi	1731	15422	89871	13737	700	40722	3296	1162	853	867	744	5	298
Azarbayejangharbi	2330	17345	11443	14485	725	37412	5059	1819	803	1666	1097	5	322
Ardebil	853	5992	5923	7060	299	17867	1640	555	262	586	477	3	148
Ostan Esfahan	5069	29865	16827	19131	512	91000	9564	3287	3552	1941	1262	17	446
Esfahan	2498	9806	7938	5066	295	16104	5409	1833	1425	1810	1030	10	215
Chaharmahal-o-bakhtiari	971	7554	4555	6296	158	16411	1651	754	257	414	313	2	109
Markazi	2215	14675	7982	11201	315	29127	4787	1625	1679	917	635	6	173
Hamedan	2118	15021	7545	9937	369	19493	3521	1667	326	1003	658	5	276
Lorestan	1797	12605	6968	8866	221	28306	2926	1158	456	917	558	3	149
Alborz	2659	12643	7211	4836	349	5142	6286	1763	1539	1886	1154	5	203
Tehran	10756	16602	22299	8469	1742	1011	20512	9482	1753	7811	4255	12	364
Ostan Tehran	7592	38199	17487	13805	720	13029	12654	3786	3648	3112	1940	26	281
Ghom	1406	5636	3569	3287	250	11237	3188	1087	711	955	481	5	83
Mashhad	2596	10707	9016	5440	396	3168	6477	2121	1516	2310	1367	11	258
Khorasanrazavi	2998	22765	12939	26143	572	103950	7613	4565	796	1552	1115	6	380
Khorasanjonobi	908	8301	4848	12197	189	151196	1539	763	267	373	327	2	150
Khorasanshomali	699	5582	4159	5763	196	28166	1205	496	221	389	310	1	103
Ahvaz	4228	12864	5531	3732	419	11304	8957	1819	988	4338	500	2	132
Khozestan	7322	34824	11633	17295	507	57945	16195	3515	1869	8066	912	2	255
Kohkiloyeh-o-boyerahmad	1083	6874	3318	4688	174	15563	1583	363	269	558	215	1	63
Zanjan	1363	8560	5437	7857	232	22164	3099	848	1429	518	390	3	149

[Downloaded from ijaor.ir on 2026-11-28]

[DOI: 10.71885/ijorlu-2023-1-619]

Ghazvin	1750	10799	4885	6763	236	15637	4412	1516	1835	753	526	4	165
Semnan	1241	7001	4202	6850	183	97491	2530	985	904	464	335	4	105
Sistan-o-balochestan	2460	19630	11369	22680	671	187502	5165	1702	147	2473	685	2	193
Kermanshah	1922	15575	6487	11221	296	24641	3142	1131	298	1072	676	2	151
Kurdestan	1273	10741	5226	9905	232	28817	2120	847	165	877	561	2	139
Illam	857	5084	2493	4367	112	20150	1320	488	74	448	193	1	63
Shiraz	3680	24105	11488	11339	530	20184	5807	2503	816	1629	920	8	202
Fars	3621	33351	11613	22059	356	103000	6977	3757	471	2105	837	5	361
Boshehr	2970	13161	5774	7100	227	23168	5486	1230	141	3257	387	2	155
Shomal-e-kerman	2019	14070	7483	11128	380	91193	4090	2172	500	922	543	3	269
Jonob-e-kerman	2708	23226	12331	18249	368	95887	5500	2679	224	1780	500	2	162
Gilan	2890	17006	18528	8648	621	14711	5080	1483	861	2063	1250	5	500
Mazandaran	3331	25825	14271	10311	624	14732	5941	1576	1111	2210	1163	9	160
Garb-e-mazandaran	1691	11287	6110	3923	232	9040	2131	706	205	899	499	3	106
Golestan	2021	14843	6988	7078	348	20381	3258	1020	414	1413	633	3	102
Hormozgan	3750	21016	8268	13926	447	66539	819	2131	242	4445	585	2	184
Yazd	1765	13385	7378	9688	349	74650	4491	1231	2092	874	569	9	226

Our proposed model (4) and three different distance function models of Kao and Hung [6] are employed on the data set of Table 4. The weights of six inputs and eight outputs are listed in Table 5. It is worth to note that, the first three columns in Table 5 (Kao and Hung [6] models) present the weights as close as to zero. In contrast, the proposed model (4) gives the weights with the least dissimilarity. Also, as the last column of Table 5 draws there is a huge discrepancy between the results of the proposed model (4) and zero.

Table 5 Common weights derived by different models

	p = 1	P = 2	P = ∞	Eff^{new}
V_1	0.00293	0.0063	0.0049	0.204230
V_2	0.0001	0.0001	0.0001	0.469186
V_3	0.0001	0.00041	0.0001	0.415018
V_4	0.0001	0.0001	0.0001	0.239133
V_5	0.00276	0.0047	0.00205	0.001598
V_6	0.0001	0.0001	0.00011	0.420507
U_1	0.0005	0.00034	0.0001	0.604785
U_2	0.0023	0.00474	0.00288	0.831806
U_3	0.0001	0.0001	0.0001	1
U_4	0.0001	0.00296	0.00321	1
U_5	0.0001	0.0001	0.0001	1
U_6	0.0001	0.0001	0.0001	1
U_7	0.0001	0.0001	0.0001	1
U_8	0.0185	0.03473	0.1907	1

To find the optimal solution to Kao and Hung's [6] Models, weight restrictions $u_r \geq \varepsilon(r=1,...,s), v_i \geq \varepsilon(i=1,..,m)$ can start with small as $\varepsilon = 0.0001$ the initial point. Turning back to Table (5), the input and output weights in the three first models of Kao and Hung [6] catches the epsilon value. Hence, the weights 'value is dependent on the epsilon value. Changing the epsilon value can reform the quantity of the weights too. On the other hand, the proposed model (4) gives all positive weights and the least dissimilarity. In order to show the advantages of our proposed model (4), we compare it with Kao and Hung [6] models and the CCR model (1)

[Downloaded from ijaor.ir on 2026-11-28]

[DOI: 10.71885/ijorlu-2023-1-619]

based on efficiency scores. Table 6 records the efficiency scores of different methods on the data set of Table (4).

The second column of Table 6 shows the CCR efficiency scores. As it can be seen, CCR model (1) shows 23 of the 39 companies as DEA efficient. The rest columns of Table (6) record the efficiency scores of Kao and Hung [6] distance functions models. There are one efficient unit in P = 1, eight efficient units in P = 2 and three efficient units in P = ∞. The efficiency scores of the proposed model (4) are recorded in the last column of Table 6. Interestingly, there is only one efficient unit in this approach. As the last row of Table 6 displays, the average of efficiency scores in proposed model (4) is the least (0.357989) among the models. The maximum average efficiency is recorded as 0.93469 in the CCR model (1). Kao and Hung [6] models with P=1 and P = ∞ have drawn the second and third rank with0.69670 and 0.731641. The next row is assigned to Kao and Hung [4] models with P=2 with the value of 0. 808232. DMU#12 is the top-ranked company in the proposed model (4). Generally, the proposed model (4) can support the advantages of the least efficient units and the least efficiency scores. Also, the proposed model (4) not only results in strictly positive weights but also can avoid weights dissimilarity and reduces the number of efficiency units.

Table 6 The efficiency scores calculated by different models

DMU	CCR	P = 1	P = 2	P = ∞	Eff^{new}
1	1	1	1	1	0.84811
2	1	0.69075	0.80934	0.701	0.18088
3	0.9930	0.79342	0.92787	0.85465	0.26452
4	1	0.73063	0.84573	0.78659	0.20834
5	1	0.66842	0.73435	0.60994	0.23862
6	1	0.96111	1	0.9378	0.54676
7	0.8824	0.70992	0.79691	0.68123	0.19888
8	0.8702	0.73714	0.79102	0.67624	0.29917
9	1	0.90926	0.99206	0.85652	0.27843
10	0.8610	0.65931	0.79523	0.69053	0.21385
11	1	0.9861	1	0.95951	0.79229
12	1	0.98946	1	0.99923	1
13	1	0.66236	065675	0.60252	0.55421
14	1	0.72485	0.79227	0.73188	0.4918
15	1	0.9892	1	0.97743	0.87614
16	1	0.81264	1	0.80582	0.1861
17	1	0.26075	0.4084	0.28697	0.03807
18	0.9261	0.55171	0.70309	0.59791	0.12408
19	1	0.76535	0.9632	1	0.93274
20	1	0.67464	0.91614	0.91825	0.48008
21	0.6250	0.51486	0.62597	0.57307	0.21351
22	1	0.73067	0.77067	0.64983	0.28877
23	1	0.89938	0.92514	0.78599	0.43252
24	0.9246	0.36329	0.5063	0.37371	0.08468
25	0.9877	0.33332	0.55507	0.49013	0.08836
26	0.7715	0.63452	0.76186	0.67884	0.2265
27	0.9233	0.58184	0.75177	0.66774	0.17178
28	0.8700	0.54577	0.703	0.60608	0.16202

[Downloaded from ijaor.ir on 2026-11-28]

[DOI: 10.71885/ijorlu-2023-1-619]

29	0.7879	0.69571	0.73849	0.66623	0.33423
30	1	0.66525	0.86281	0.70802	0.16466
31	1	0.7576	1	0.99733	0.44553
32	1	0.65895	0.84585	0.6557	0.13007
33	0.9033	0.53301	0.69768	0.59402	0.13614
34	1	0.99528	1	1	0.37024
35	0.7377	0.68044	0.74571	0.73164	0.35526
36	0.6618	0.55465	0.61504	0.59267	0.27535
37	0.7277	0.53269	0.65197	0.63335	0.26702
38	1	0.62215	0.90364	0.87635	0.2882
39	1	0.86497	0.72151	0.57929	0.17722
Average	0.934697	0.6967018	0.8082326	0.7316413	0.3579869

5 Conclusions

In this paper, an alternative approach has been proposed to address the issue of common set of weights (CSW) in DEA literature. The key point of this method is to minimize deviations of real inputs and outputs of units and the virtual inputs and outputs composed of extreme efficient units. The proposed model has several contributions in study of common set of weights (CSW). First, the proposed model result in strictly positive weights and avoids weight dissimilarity simultaneously. Second, the proposed model reduced the efficiency scores and the number of the efficient units. The application illustration revealed that the proposed models can support the idea of choosing dissimilar and positive
Common set of weights.

References

1. Ganley, J.A., Cubbin, S.A. (1992). Public sector efficiency measurement: Applications of data envelopment analysis. Amsterdam: North-Holland.
2. Roll, Y., Cook, W.D., Golany, B. (1991). Controlling factor weight in data envelopment analysis. IIE Transactions, 23, 2-9.
3. Roll, Y., Golany, B. (1993). Alternative methods of treating factor weights in DEA. Omega, 21, 99-109.
4. Sinuany – Stern, Z., Mebrez, A., Burbog, A. (1994). Academic departments, efficiency in DEA. Computers and Operations Research. 21, 543 -556.
5. Sinuany-Stern, Z., Friedman, L. (1998). DEA and the discriminant analysis of ratios for ranking units. European Journal of Operation Research, 111, 470 – 478.
6. Kao, C., Hung, H.T. (2005). Data envelopment analysis with common weights: the compromise solution approach. Journal of Operational Research Society. 56, 1199-1203.
7. Liu, F.H. F., Peng, H.H. (2008). Ranking of units on the DEA frontier with common weights. Computers and Operations Research. 35(5), 1624-1637.
8. Wang, Y.M., Luo, Y., Liang, L. (2009). Ranking decision-making units by imposing a minimum weight restriction in the data envelopment analysis. Journal of Computation and Applied Mathematics, 223, 469-484.
9. Wang, Y.M., Luo, Y., LAN, Y.X., (2011). Common weights for fully ranking decision-making units by regression analysis. Expert System with Application. 38, 9122 – 9128.
10. Sun, J., Wu, J., Gue, D. (2013). Performance ranking of units considering ideal and anti-ideal DMU with common weights. Applied Mathematical Modelling 37, 6301 – 6310.
11. Charnes, A., Cooper, W.W., Rhodes, E. (1978). Measuring the efficiency of decision-making units. European Journal of Operation Research. 2, 429-444.

[Downloaded from ijaor.ir on 2026-11-28]

[DOI: 10.71885/ijorlu-2023-1-619]

12. Lovell, C.A.K., Pastor, J. T. (1995). Units invariant and translation invariant DEA models. Operations Research Letters. 18, 147-151.
13. Pourhabib, A., Kordrostami, S., Amirteimoori, A., Kazemi matin, R. (2018). Performance measurement in data envelopment analysis without slacks: An application to electricity distribution companies. RAIRO Operation Research, 52, 1069-1085.
14. Shang, J., Sueyoshi, A. (1995). A unified framework for the selection of flexible manufacturing systems. European Journal of Operation Research, 85, 297-315.
15. Yuzhi, S. and Zhangna, Y. (2012). Study of the Input-Output Overall Performance Evaluation of Electricity Distribution Based on DEA Method. Energy Procedia. 16, 1517-1525.
16. Çelen, A. (2013). Efficiency and productivity (TFP) of the Turkish electricity distribution companies: An application of two-stage (DEA& TOBIT) analysis. Energy Policy, 63, 300-310.
17. Omrani, H., Gharizadeh –Beiragh, R., ShafieiKaleibari S. (2015), Performance assessment of Iranian electricity distribution companies by an integrated cooperative game data envelopment analysis principal component analysis approach. International Journal of Electrical Power & Energy Systems, 64, 617-625.
18. Tavassoli M., Faramarzi G. R., Farzi Poor Saen R., (2015). Ranking electricity distribution units using slacks-based measure, strong complementary slackness condition, and discriminant analysis, International Journal of Electrical Power & Energy Systems. 64, 1214-1220.

[Downloaded from ijaor.ir on 2026-11-28]

[DOI: 10.71885/ijorlu-2023-1-619]